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April 29, 2014

Exam 9

Financial Risk and Rate of Return

INSTRUCTIONS TO CANDIDATES

1. This 53.5 point examination consists of 24 problem and essay questions.

2. For the problem and essay questions, the number of points for each full question and part of a question is indicated at the beginning of the question or part. Answer these questions on the lined sheets provided in your Examination Envelope. Use <u>dark</u> pencil or ink. Do not use multiple colors or correction fluid.

- Write your Candidate ID number and the examination number, 9, at the top of each answersheet. Your name, or any other identifying mark, must not appear.

- Do not answer more than one question on a single sheet of paper. <u>Write only on the front lined side of the paper</u>– **DO NOT WRITE ON THE BACK OF THE PAPER.** Be careful to give the number of the question you are answering on each sheet. If your response cannot be confined to one page, please use additional sheets of paper as necessary. Clearly mark the question number on each page of the response in addition to using a label such as "Page 1 of 2" on the first sheet of paper and then "Page 2 of 2" on the second sheet of paper.

- The answer should be concise and confined to the question as posed. <u>When a specified number of items are requested, do not offer more items than requested.</u> For example, if you are requested to provide three items, only the first three responses will be graded.

- <u>In order to receive full credit</u> or to maximize partial credit on mathematical and computational questions, you must clearly outline your approach in either verbal or mathematical form, <u>showing calculations</u> where necessary. Also, you must clearly <u>specify any additional assumptions</u> you have made to answer the question.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

4. Prior to the start of the exam, you will have a **fifteen-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. A chart indicating the point value for each question is attached to the back of the examination.Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils.You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.

   • Verify that the table of the Normal Distribution is attached to the examination after the last question.

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until two hours after the start of the examination. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.

7. At the end of the examination, place all answer sheets in the Examination Envelope. Please insert your answer sheets in your envelope in question number order. Insert a numbered page for each question, even if you have not attempted to answer that question. Nothing written in the examination booklet will be graded. Only the answer sheets will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by May 14, 2014.

END OF INSTRUCTIONS

1.   (1.25 points)

Given the following about a risky portfolio, P:

*   Expected return of P is 16%.
*   Standard deviation of expected return of P is 24%.
*   Borrowing rate is 10%.
*   Lending rate is 6%.

Graph the capital allocation line (CAL). Plot the position of P on the CAL graph. Clearly label the axes, the y-intercept of the CAL, and the borrowing and lending ranges.

2.   (1.5 points)

Contrast active and passive investment strategies regarding:

i.   constructing a portfolio.
ii.   implementation and total costs.
iii. expected return.

3.   (2 points)

The values in the table below were empirically estimated.

Security	Expected Return	Beta
Stock A	6%	0.5
Stock B	10%	1.5

- The expected market return is 8%.
- The risk-free rate is 2%.

a.   (1.5 points)

On a single graph, draw and label:

    i.   The Security Market Line (SML) implied by the expected market return.
    ii.   The empirically estimated SML.
    iii. The risk-return point for both securities listed above.

b.   (0.5 point)

Explain whether Stock A or Stock B is a better buy according to the Capital Asset Pricing Model (CAPM).

4.   (1.5 points)

a.   (1 point)

Discuss two challenges with using a Markowitz model to analyze 200 technology stocks within a broader portfolio.

b.   (0.5 point)

Briefly discuss how the single-index model does or does not overcome each of the challenges of the Markowitz model identified in part a. above.

5.   (2.75 points)

Given the following information for constructing an investment portfolio:

Asset	Expected Return	Standard Deviation
A	8%	20%
B	10%	25%

- The risk-free rate is 2%.
- The correlation coefficient between assets A and B is -0.4.
- The investor has a risk aversion coefficient of 5.

Calculate the expected rate of return and standard deviation of the optimal complete portfolio.

6. (2 points)

Given the following information:

- $\beta_{Market} = 1.0$
- $\beta_{SMB} = 0.8$
- $\beta_{HML} = 1.1$
- $\alpha_{Stock\ A} = 2\%$

Portfolio characteristic	Expected Return
Small capitalization stocks	9%
Medium capitalization stocks	7%
Big capitalization stocks	5%
High book-to-market ratio stocks	7%
Medium book-to-market ratio stocks	5%
Low book-to-market ratio stocks	4%
Market	6%

- The risk-free rate is 1%.
- Assume no firm-specific influences.

a. (0.5 point)

Briefly describe two market anomalies exhibited by the returns of the portfolios that seem to contradict the semi-strong form of the efficient market hypothesis.

b. (1 point)

Calculate the expected return for Stock A using the Fama-French three-factor model.

c. (0.5 point)

Describe the impact of liquidity on the Fama-French three-factor model.

7.   (1.75 points)

Given the following information about bonds with coupons paid annually:

Bond	Bond Par Value	Time to Maturity (in years)	Annual Coupon Rate	Bond Price
1	$100	1.0	0%	$95.12
2	$100	2.0	0%	Unknown
3	$100	3.0	0%	$78.66
4	$100	3.0	5%	$91.72

Company A and Company B enter into a forward rate agreement with the following attributes:

- Company A will lend money to Company B in two years for a term of one year.
- The principal amount of this loan is $8,000,000.
- The annual interest rate for this loan is 12% compounded annually.

Calculate the value of the forward rate agreement to Company B.

8.   (2 points)

Given the following information below regarding a series of bonds:

Bond	Maturity (Years)	Face Value	Annual Coupon
A	1.0	$1000	0%
B	2.0	$1000	0%
C	3.0	$1000	3%
D	3.0	$1000	0%

- The price of bond A is $951.23.
- The two-year spot rate is 8%, continuously compounded.
- The price of bond C is $901.78.

a.   (1.5 points)

Plot the pure yield curve.

b.   (0.5 point)

Explain the shape of the pure yield curve plotted in part a. above using a term-structure theory of interest rates.

9.   (2.5 points)

Companies A and B both want to borrow $50 million for 10 years and are offered the following annual borrowing rates:

	Floating	Fixed
Company A	LIBOR – 0.2%	4.3%
Company B	LIBOR + 0.9%	5.4%

a.   (0.5 point)

Explain why the comparative advantage argument does not provide a benefit to this pair of companies when borrowing from the floating and fixed interest rate markets.

b.   (2 points)

Assume instead that the following interest rates are available for each company:

	Floating	Fixed
Company A	LIBOR – 0.1%	4.1%
Company B	LIBOR + 0.9%	5.4%

Devise a swap agreement that is mutually beneficial to both companies and creates a 0.06% gain to a financial institution serving as an intermediary.

10. (2 points)

Given the following information regarding an insurance company:

- Surplus is $65 million.
- Annual client retention is 88%.
- Premiums are collected and expenses are paid at the beginning of each year.
- Losses are paid at the end of each year.
- The current target return on surplus follows the formula $k = a + by$, where $a = 12.0\%$, $b = 0$, and y is the risk-free interest rate of 5.0%.

a. (0.5 point)

Calculate the franchise value of the firm.

b. (0.5 point)

Calculate the duration of the franchise value of the firm.

c. (1 point)

Assume the following are two alternative pricing strategies:

Component	Strategy 1	Strategy 2
a	7.0%	10.0%
b	1.0	0.5
Franchise Value ($000)	$22,430	$24,030
Duration of Franchise Value	6.83	13.50

Make a recommendation on which alternative strategy to pursue and support the recommendation with information provided.

11.   (2.5 points)

Given the following:

Coupon Bonds

Term	Yield	Duration (Years)
4-year	7%	3
10-year	9%	7
20-year	10%	11
30-year	11%	20

Common Stocks

Stocks	Annual Dividend	Duration (Years)
A	5.00%	20
B	9.30%	11
C	10.00%	10
D	14.30%	7

Zero Coupon Bonds

Term	Yield
3-year	1.50%
5-year	2.00%
7-year	2.50%
10-year	3.00%
15-year	3.50%
20-year	4.00%

An insurance company has liabilities with the following payment pattern:

Development Year	Payment
0 to 2	0
3	2.8 million
4 to 6	0
7	5.2 million
8 to 19	0
20	7.0 million

- The liability discount rate is 3.0%.
- Assume all payments occur at the end of the year.

a.   (0.5 point)

Calculate the Macaulay Duration of the liability.

b.   (1.5 points)

The target rate of return on investments is 11.6%.

Using the securities listed above, design an investment portfolio that is immunized from small changes in interest rates and has an expected return greater than or equal to the target rate of return on investments.

c.   (0.5 point)

Explain which investments from those listed above should be included in the portfolio to be completely immunized from all interest rate changes.

12. (2.75 points)

A company's capital allocation will be based on a simulation of 1,000 underwriting outcomes for the three lines of business that the company writes.

The company will use proportional allocation based on one of the following two risk measures:

- 99.5% conditional tail expectation (CTE).
- 99.5% value at risk (VaR).

The 10 worst loss scenarios for each line, sorted independently, are in the table below.

Line A		Line B		Line C		Total	
Rank	Loss	Rank	Loss	Rank	Loss	Rank	Loss
1	1,000	1	10,000	1	1,500	1	11,500
2	985	2	9,000	2	1,000	2	9,000
3	980	3	8,800	3	975	3	9,500
4	925	4	7,900	4	950	4	8,700
5	850	5	4,000	5	940	5	7,000
6	650	6	2,000	6	750	6	2,400
7	500	7	1,000	7	675	7	1,190
8	495	8	950	8	600	8	1,185
9	490	9	900	9	500	9	1,165
10	485	10	800	10	450	10	1,150

a. (1.25 points)

Calculate the allocation percentages for each line using both risk measures.

b. (1 point)

The company is primarily concerned with holding enough capital to cover the magnitude of a shortfall event. Contrast how well each of the allocation bases will address the company's primary concern.

c. (0.5 point)

Describe the impact to the amount of capital allocated to line B if the company were to use a CTE co-measures approach at the 99.5% level.

13. (2 points)

Federal National Mortgage Association (Fannie Mae) and Federal Home Loan Mortgage Corporation (Freddie Mac) introduced the following innovations to the mortgage market:

- Purchasing whole mortgages from issuers.
- Issuing pass-through securitized mortgages.

For each of these mortgage innovations:

a. (1 point)

Describe the market issue it addressed.

b. (1 point)

Describe an analogous insurance risk transfer mechanism and the market issue it addresses.

14.   (2 points)

A company writes two lines of business, A and B. The claim payment patterns are as follows:

End of Year	A	B
1	60%	80%
2	30%	20%
3	10%	

Assume the following:

- Risk-free interest rate is 5%.
- Cost of capital is 15% for each line.
- Undiscounted loss ratio is 70% for each line.
- Expense ratio is 25% for each line.
- Premium is $43 million for each line.
- Each line has been assigned $25 million of risk capital at the beginning of year 1.
- Expenses are paid at the end of year 1, and capital is released as claims are paid.

Determine which line of business the company should grow using the risk-adjusted return on capital (RAROC) framework.

15. (2 points)

The following credit ratings transition matrix applies to bonds initially rated A or better:

Initial Rating	Rating at Year-End			
	Aaa	Aa	A	Baa or worse
Aaa	0.95	0.04	0.01	0.00
Aa	0.05	0.90	0.04	0.01
A	0.00	0.05	0.90	0.05

The ratings transitions for two bonds can be correlated using a Gaussian copula with correlation 0.3.

A company's investment portfolio consists of two bonds:

- One bond initially rated Aa
- One bond initially rated A

The company's actuary samples 1,000,000 pairs of random variables (Z_1, Z_2) from a bivariate normal distribution with correlation coefficient 0.3. The table below summarizes the number of times the sampled pairs were within the indicated ranges:

Range of Z_1 Values	Range of Z_2 Values					
	< -2.326	-2.326 to -1.645	-1.645 to 0.000	0.000 to 1.645	1.645 to 2.326	> 2.326
< -2.326	556	1,261	6,091	1,967	48	6
-2.326 to -1.645	1,333	3,895	23,702	10,489	400	50
-1.645 to 0.000	6,075	23,797	231,668	175,999	10,539	1,971
0.000 to 1.645	2,039	10,746	175,772	231,333	23,968	6,160
1.645 to 2.326	68	458	10,967	23,371	3,667	1,275
> 2.326	7	68	2,059	6,259	1,366	570

Produce a range of estimates for the probability that both bonds in the portfolio will be downgraded to Baa or worse by year-end.

16.   (3.5 points)

Consider three securities:

- A catastrophe bond.
- A junior tranche on a collateralized debt obligation (CDO).
- A senior tranche on a collateralized debt obligation squared (CDO^2).

Assume the following:

- The market estimates the same probability of default for the catastrophe bond as for the individual mortgages underlying the CDO and CDO^2.
- The market expects the same return on all three securities.

a.   (0.5 point)

Explain which of the three securities would be considered preferable according to the capital asset pricing model (CAPM).

b.   (1.5 points)

Assume that the probability of default on the underlying mortgages and the catastrophe bond increase by the same amount. Explain how an investor should rank the three securities from most to least preferable.

c.   (1.5 points)

Assume the correlation of defaults for the underlying mortgages moderately increases. Explain the impact on the market price for each of the three securities.

17.   (1.75 points)

The board of directors for a company is deciding between two incentive plans for the company's management:

- Stock options
- Direct ownership of shares

a.   (1 point)

Discuss how the board's decision might impact the company's focus on risk management.

b.   (0.75 point)

Explain which plan better aligns management incentives with the interest of shareholders if the company is already in the midst of severe financial distress but not yet in bankruptcy.

18. (3 points)

Assume the following for a publicly traded personal lines insurance company:

- Premium is $125,000,000.
- Expense ratio is 28%.
- Premium is collected and expenses are paid at policy inception.
- Loss ratio is 75%.
- 75% of losses are paid at the end of the first year; the remainder at the end of the second year.
- The company allocates surplus to maintain a 3:2 undiscounted loss reserve to surplus ratio.
- Cost of capital is 7%.
- Investment yield is 4%.
- Risk-free rate is 3.5%.

a. (2.5 points)

Calculate the internal rate of return.

b. (0.5 point)

Discuss the adequacy of the rates from the perspective of equity providers.

19.   (2.25 points)

An insurance company is reviewing a new line of business using the present value offset method for calculating the underwriting profit provision. Expected payment patterns for the reference line and the reviewed line are as follows:

Year	Reference Line of Business	Reviewed Line of Business
1	50%	10%
2	30%	40%
3	20%	30%
4	0%	20%

- All loss payments are made at the end of the year.
- The underwriting profit provision for the reference line of business is 5.0%.
- The underwriting profit provision for the reviewed line of business is 2.5%.
- The historic investment rate of return is 6.5%.
- The new money rate of return is 4.0%.

a.   (1.25 points)

Calculate the permissible loss ratio for the reviewed line of business.

b.   (1 point)

The present value offset method was used to determine the underwriting profit provision for rates filed in a state. The state's insurance department responds with the following:

1. Please indicate what discount rate is used in the calculation of the underwriting profit provision and if a discount rate was used that is less than the investment rate of return, justify why this is acceptable.

2. Please provide the target return on surplus that was used to generate the filed rates.

Respond to both of these items in support of the rates filed.

20.   (2.5 points)

a.   (1.5 points)

Identify and briefly describe three types of underwriting profit.

b.   (1 point)

Formulate an argument for why the underwriting profit provision should include the investment income earned on stockholder supplied funds.

21.   (1 point)

Describe why allocation of surplus by state and line using premium-to-surplus ratios may be problematic for each of the following:

  i.   An individual multi-line national insurer.
  ii.   The insurance industry as a whole.

22.   (4.25 points)

ABC Reinsurance Company is considering whether to write one of the following portfolios of property exposures:

- Portfolio A contains 100 locations with a combined total insured value of 45.
- Portfolio B contains 250 locations with a combined total insured value of 160.

Below is the distribution of events and corresponding loss amounts from a catastrophe model:

Event	Prob p(i)	ABC Reinsurance Company	Portfolio A	Portfolio B	ABC + Portfolio A	ABC + Portfolio B
1	0.010	350.00	2.00	150.00	352.00	500.00
2	0.020	200.00	0.50	50.00	200.50	250.00
3	0.010	1.00	40.00	2.50	41.00	3.50
4	0.030	75.00	0.40	24.00	75.40	99.00
5	0.015	160.00	1.40	80.00	161.40	240.00
Average Annual Loss (Mean)		12.16	0.46	4.45	12.62	16.61
Variance		2,538.69	15.92	383.13	2,581.54	4,836.37
Coefficient of Variation		4.14	8.62	4.40	4.03	4.19
Profit		3.20			3.38	4.28
Total Risk Adjusted Capital		20.00			22.00	25.00

Below is the covariance share table between ABC Reinsurance and Portfolio A:

Event	ABC	Portfolio A
1	Unknown	0.079
2	3.910	0.010
3	0.019	Unknown
4	1.737	0.009
5	6.562	0.057

- ABC Reinsurance Company's share of the covariance with Portfolio B is 1,372.788.
- Portfolio B's share of the covariance with ABC Reinsurance Company is 541.762.
- The risk load multiplier, λ is 0.005.
- ABC Reinsurance Company only has enough capital to write one portfolio.
- ABC Reinsurance Company requires a 15% RAROC after adding the new portfolio.

<QUESTION 22 CONTINUED ON NEXT PAGE>

Question 22 continued:

a.   (2.25 points)

Using the covariance share method, calculate the renewal risk loads to complete the table below:

Scenario	Risk Load Allocated To		
	ABC	Added Portfolio	Total
ABC + Portfolio A			
ABC + Portfolio B			

b.   (2 points)

Using the information provided and outputs from the solution to part a. above, fully discuss the elements ABC Reinsurance Company should consider when deciding which portfolio to write.

23.   (1.75 points)

A reinsurance contract is expected to make a single loss payment at the end of one year.

Given the following information:

Loss distribution	Normal
Expected loss	$4,000,000
Standard deviation of loss	$6,000,000
Risk-free rate	4%
Target yield	12%
Standard deviation of yield rate	15%
Normal Table z-Score at 99.9th Percentile	3.0902

Using the swap method, calculate the assets required for the reinsurer to cover a loss at the 99.9th percentile.

24.   (3 points)

Industry data for a single line of business over a three year period is as follows.

Year	Net Written Premiums	Loss & Expense Reserves
1	$100,000,000	$50,000,000
2	$102,000,000	$53,550,000
3	$104,040,000	$57,352,050

- Net written premium and loss and expense reserves are stated on a nominal basis
- Inflation is 2% annually
- Surplus is adjusted at the beginning of each year to support business expected to be written in the coming year

a.   (2 points)

Calculate the required percentage change in surplus for the coming year.

b.   (1 point)

A publicly traded insurance company has $20 million of surplus at end of year 3 and will:

- Raise $1,000,000 in paid-in-capital.
- Pay $1,500,000 in shareholder dividends.

Calculate the required rate of return for this insurance company.

Exam 9
Financial Risk & Rate of Return

Point Value by Question Grid

Question	Total Points	Percent of Total	a.	b.	c.
1	1.25	2.3%			
2	1.50	2.8%			
3	2.00	3.7%	1.50	0.50	
4	1.50	2.8%	1.00	0.50	
5	2.75	5.1%			
6	2.00	3.7%	0.50	1.00	0.50
7	1.75	3.3%			
8	2.00	3.7%	1.50	0.50	
9	2.50	4.7%	0.50	2.00	
10	2.00	3.7%	0.50	0.50	1.00
11	2.50	4.7%	0.50	1.50	0.50
12	2.75	5.1%	1.25	1.00	0.50
13	2.00	3.7%	1.00	1.00	
14	2.00	3.7%			
15	2.00	3.7%			
16	3.50	6.5%	0.50	1.50	1.50
17	1.75	3.3%	1.00	0.75	
18	3.00	5.6%	2.50	0.50	
19	2.25	4.2%	1.25	1.00	
20	2.50	4.7%	1.50	1.00	
21	1.00	1.9%			
22	4.25	7.9%	2.25	2.00	
23	1.75	3.3%			
24	3.00	5.6%			
Total	53.50	100.0%			

Tables of the Normal Distribution

Probability Content
from -∞ to Z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Values of z for selected values of Pr(Z<z)

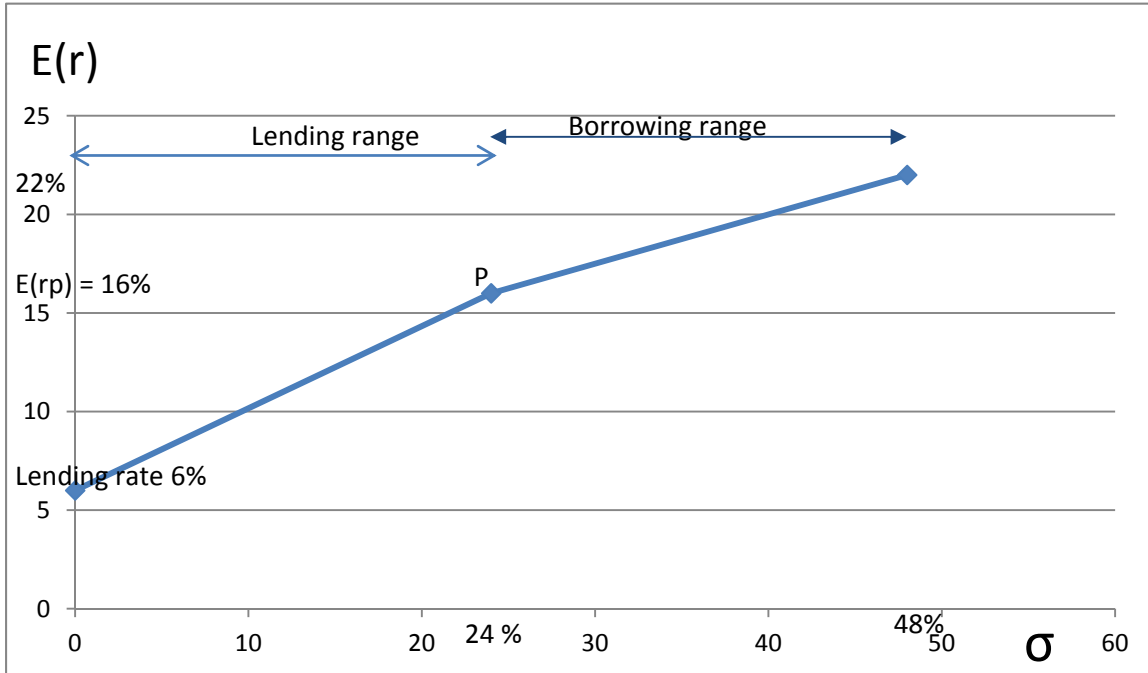| z	0.842	1.036	1.282	1.645	1.960	2.326	2.576
Pr(Z<z)	0.800	0.850	0.900	0.950	0.975	0.990	0.995

GENERAL COMMENTS:

- Candidates should note that the instructions to the exam explicitly say to show all work; graders expect to see enough support on the candidate's answer sheet to follow the calculations performed. While the graders made every attempt to follow calculations that were not well-documented, lack of documentation may result in the deduction of points where the calculations cannot be followed or are not sufficiently supported.
- Incorrect responses in one part of a question did not preclude candidates from receiving credit for correct work on subsequent parts of the question that depended upon that response.
- Candidates should try to be cognizant of the way an exam question is worded. They must look for key words such as "briefly" or "fully" within the problem. We refer candidates to the Future Fellows article from December 2009 entitled "The Importance of Adverbs" for additional information on this topic.
- Some candidates provided lengthy responses to a "briefly describe" question, which does not provide extra credit and only takes up additional time during the exam.
- The opposite can be said for questions that said "describe" or "fully describe." For these many candidates wrote far too little when the sub-part was worth 1+ point (e.g., Q22, part b. which was worth 2 points, many papers had one or two sentences as the response). Let the key word AND question point value be your guide on how much to write for an essay question.
- Generally, candidates were fairly well prepared for this exam, see below for exam statistics. However, candidates should be cautious of relying solely on study manuals, as some candidates lost credit for failing to provide basic insights that were contained in the syllabus readings.

EXAM STATISTICS:

- Number of Candidates: 418
- Available Points: 53.5
- Passing Score: 38.25
- Number of Passing Candidates: 201
- Raw Pass Ratio: 48.09%
- Effective Pass Ratio: 50.12%

QUESTION *#1*	
TOTAL POINT VALUE: *1.25*	**LEARNING OBJECTIVE:** *A.1 Explain key concepts of risk: Appetite, Tolerance, Aversion, Measurement, Portfolio Construction, Strategies for Monitoring*
SAMPLE ANSWERS	
Part a: *1.25* point(s)	

expected model solution overall

E(r)

25

22%

20

E(rp) = 16%

15

10

Lending rate 6%

5

0

Lending range   Borrowing range

P

0   10   20   24 %   30   40   48%   50   σ   60

lend/borrow labeled horizontally instead of vertically

E(N_i)

0.25

0.2

0.15

0.1

0.05

0

P (.24, .16)

(0,.06)

Borrowing range
E(N_i) > .16

Lending range
.06 < E(N_i) < .16

0   0.1   0.2   σ_i   0.3   0.4   0.5   0.6
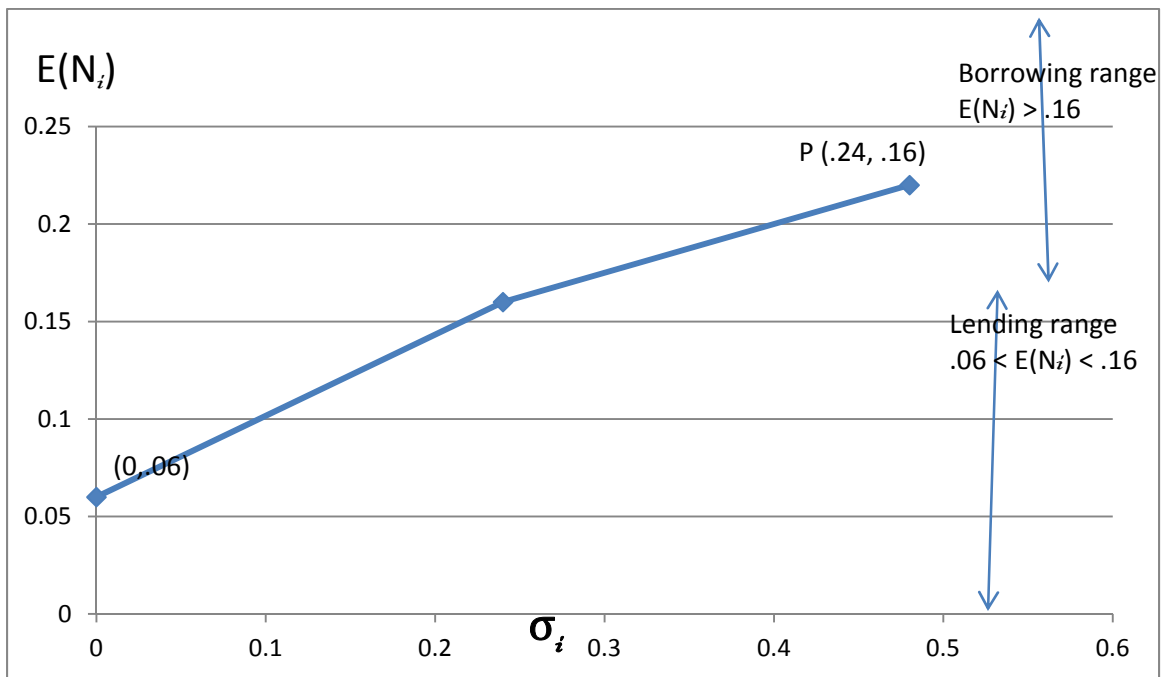
lend/borrow identified by calculating lend/borrow slopes above (and clearly labeled as such) and then labelling the slopes of the line segments on the graph

$$S_{\text{Lend?}} = \frac{.16-.06}{.24} = \frac{Er-rf}{\sigma} = .4167$$

$$S_{\text{borr}} = \frac{.16-.10}{.24} = .25$$
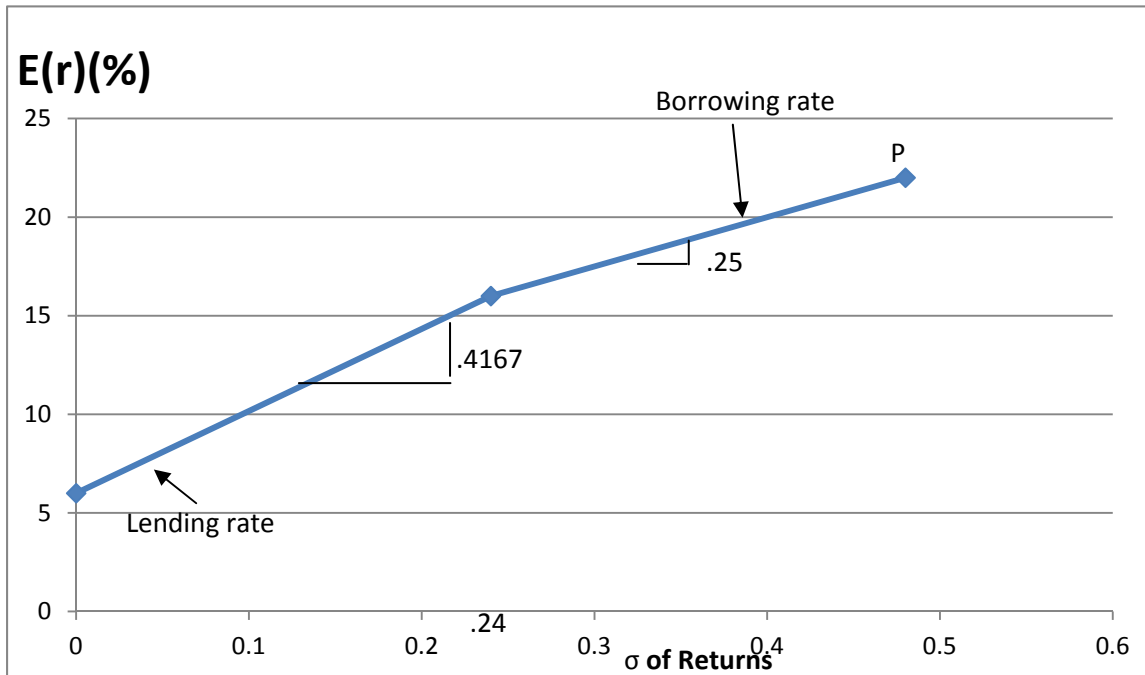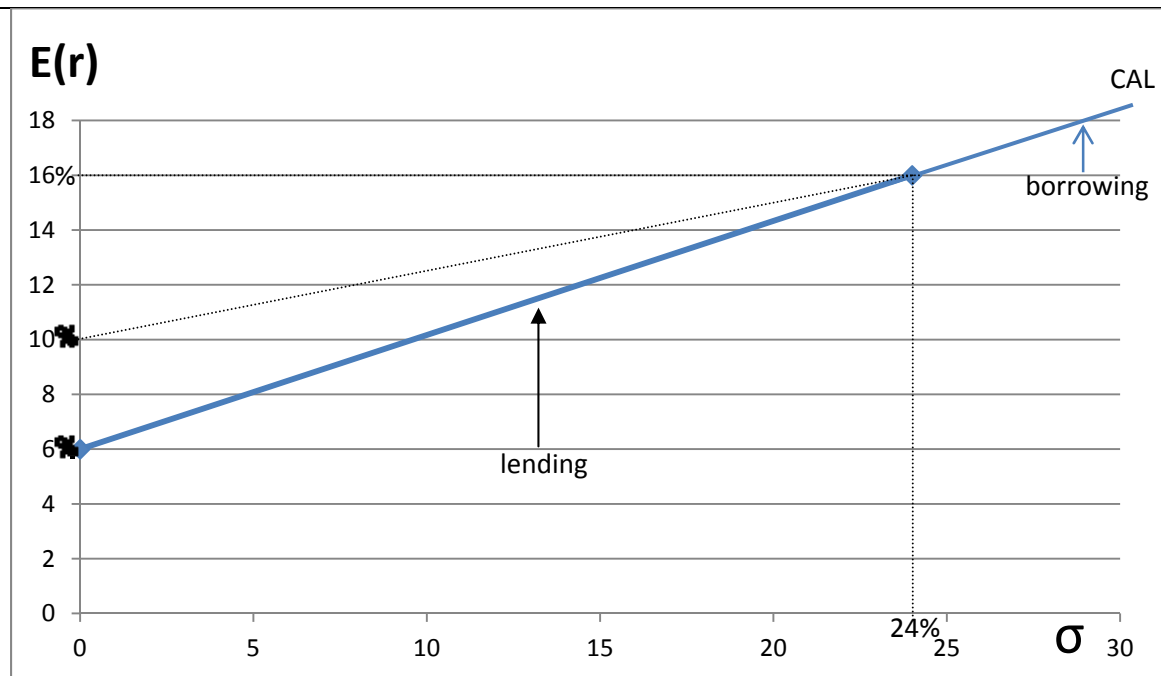
P not labeled in graph, but coordinates clearly stated below

$E(r_p)$ = 16%, σ_p = 24%, r_B = 10%, r_f = 6%

Sharpe ratio for lending: $\dfrac{E(r_P)-r_f}{\sigma_P} = \dfrac{16\%-6\%}{24\%} = 0.4167$

Sharpe ratio for borrow: $\dfrac{E(r_P)-r_B}{\sigma_P} = \dfrac{16\%-10\%}{24\%} = 0.25$

EXAMINER'S REPORT

General Commentary

The candidate was expected to know:

- How to graph a CAL: axes, y-intercept, general idea of slope (without computation of slope)
- Identification of the optimal portfolio P on the CAL
- To understand that a CAL will be "kinked" – when the portfolio is invested at the risk free rate of 6% then the investor is technically lending money up until the point of the optimal portfolio P; past the optimal portfolio point, in order to invest more money, they must borrow at the borrowing rate 10%; this causes the slope to kink down given the higher y-intercept of 10%

To obtain full credit, the candidate needed to do the following:

- Draw a graph of the CAL, including the following
- Correct labeling of the axes – x-axis (standard deviation) & y-axis (E(r) not r)
- Illustrate that the CAL is kinked (didn't need to give exact calculations of slope, but needed to illustrate it is kinked down after P) at point P
- Identify P and label the coordinates
- Have the CAL going through the correct y-intercept of 6%
- Correctly label the lending (0 to P) and borrowing ranges (P, infinity)

Common errors:
- Candidates did not label P
- Candidates did not give the correct coordinates of P
- Candidates did not kink the CAL
- Candidates kinked the CAL at the wrong point
- Candidates forgot to label lending/borrowing
- Candidates labeled the wrong ranges for lending/borrowing
- Candidates labeled y-axis as r instead of E(r)
- Candidates were unclear in what the CAL was – for example, multiple solid lines
- Candidates often spent too much time calculating extra information that was not necessary (e.g., slopes) and writing explanations instead of just labeling

QUESTION *2*	
TOTAL POINT VALUE: *1.5*	**LEARNING OBJECTIVE: *A - 1***
SAMPLE ANSWERS	

1.5 points

There were three parts to our question, and we allocated 0.5 to each of these individual parts. Within each part, the 0.5 was broken down further into two 0.25 pieces (0.25 for an answer regarding the active strategy and 0.25 for an answer regarding the passive strategy)

i)   Active: Active management seeks to discover stocks with positive alphas, so will put more weight on stocks which analysts think are underpriced. Passive: Seeks to replicate a well-diversified portfolio such as the S&P 500. No effort is made to select stocks; b/c it assumes that it will cost too much for the expected benefits.

Since a passive strategy is much easier to construct. The investor may find an index fund that matches his investment risk appetite and is done (say a fund that follows the S&P 500). Active strategies on the other hand involve hand picking individual investments. The investor will need research and knowledge and time to construct that portfolio, or hire someone.

With a passive strategy, there is no need to construct a portfolio as we are simply following a selected market index. With an active strategy, you have to select the securities you want to invest in, so there is much more work there.

Active-involves actively seeking over & under-priced securities to exploit mispricings. Passive-takes prices as given, does not seek to actively analyze securities. Instead focus on well diversified portfolio, often replicating a market index.

Constructing a portfolio: An active strategy requires analysis of stock prices in an effort to seek out mis-priced stocks and add underpriced stocks to the portfolio while removing or adding short positions on overpriced stocks. Analysis may be technical or fundamental. On the other hand, a passive strategy would invest in the overall market or a market index as the risky portfolio and adjust only the amounts invested in the risky portfolio and the risk-free asset based on the investors risk appetite.

ii)   An active portfolio involves lots of on-going trading which incur fees. A passive portfolio might incur some cost to set up initially, but then there is minimal additional cost. Even investing in a professionally managed fund like Vanguard that mimics a market index has substantially less fees than a fund that practices active trading.

Active strategy is much more expensive. It requires trading often. So transaction costs are higher. Besides, efforts need to put to find the mispriced stocks. This adds additional costs to total cost. Passive strategy is much cheaper. It doesn't need to trade often so transaction costs are low. Besides, no research is needed => total cost is lower.

The active strategy will have significantly higher costs.

              -must spend significant time and resources to indentify mispriced securities
              -will need to continually analyze portfolio in order to make adjustments if security returns change over time

The passive strategy will be less costly because the investor simply chooses a well diversified portfolio and then makes very few adjustments. For example, could invest in mutual fund. Would only pay a small service fee.

Costs – Active managers will incur high implementation costs in constructing the portfolios, and will continue to incur high costs as it manages the portfolio. Passive investors have the free-rider benefit, incurring no costs to manage the index.

Implementation and total costs: An active strategy will cost much more than a passive strategy. There are costs of doing the analysis which apply both at implementation and as long as the portfolio is held and there are transaction costs each time the portfolio is changed. The passive strategy, however, is relatively cheap as there is no need for analysis and minimal transaction cost.

iii)   If the markets are efficient, it will be difficult to beat the market over a long period of time. This would nudge you to choose passive investing to save on the costs. Active investors are likely to have some very good and some very bad years. Expenses will result in lower returns.

Passive simply earns the market index return, while active strategies will earn an additional profit if they accurately identify non-zero alpha stocks -> but this will be offset by transaction costs and other investment expenses. If markets are indeed efficient (debateable), active strategies will not outperform passive strategies.

Expected return: The two strategies are likely to provide similar expected return. On average, it is not possible to outperform the market as the market is all investors. Any excess returns earned through an active strategy are likely to be offset by the associated costs.

EXAMINER'S REPORT

i.   The majority of the students acknowledged the differences between constructing an active portfolio versus a passive portfolio. The candidate needed to demonstrate that the active portfolio construction requires more work/analysis versus the passive. The most common answers we saw that received full credit mentioned that with the active strategy you are selecting individual stocks or looking for mispriced securities, while for the passive strategy you simply hold a market portfolio (i.e. the S&P 500) or select a fund that mimics the market. Only partial credit was awarded if the candidate answered for the passive piece and not the active, or vice versa. Partial credit was also awarded here if they did not fully contrast the two investment types.

ii.   We accepted a variety of answers for part ii of this question. In order to receive full credit candidates had to mention transaction costs are higher for active because it involves actively buying and selling securities, while the passive have lower transaction costs because they are buying and holding. We also awarded credit if the candidate wrote that costs are higher for active due to significant research for security analysis to find mispriced securities. Lastly, we awarded credit if the candidate mentioned the "free rider benefit" and how it results in lower costs for the passive investor since they can rely on the active investors to do the security analysis and they just buy and hold. A couple common themes where only partial credit was awarded were if the candidate did not elaborate well enough and we would have to make assumptions, or if only definitions were given without contrasting (example: Passive = Low Cost, Active = High Cost).

iii. Part iii had the most variation in answers. Credit was awarded if you specified if the passive strategy would be expecting a return to similar to that market, and while the active strategy expects a higher return, history shows that this is not always achieved.   In order to receive full credit, the candidate needed to acknowledge that the active strategy does not necessarily outperform the passive strategy because the high transaction costs associated with the active will lower its return.

QUESTION *3*	
TOTAL POINT VALUE: *2*	**LEARNING OBJECTIVE: *A.6***
SAMPLE ANSWERS	

Part a: *1.5* point(s)

Part a: 1.5 points

Sample 1:

SML $\qquad$ $E(r_i) = r_f + \beta(E(r_m) - r_f)$

$\quad E(r_m) = .08$

$\quad r_f = .02$

$\qquad B = 1 \quad \text{for} \quad r_m$

E(r)
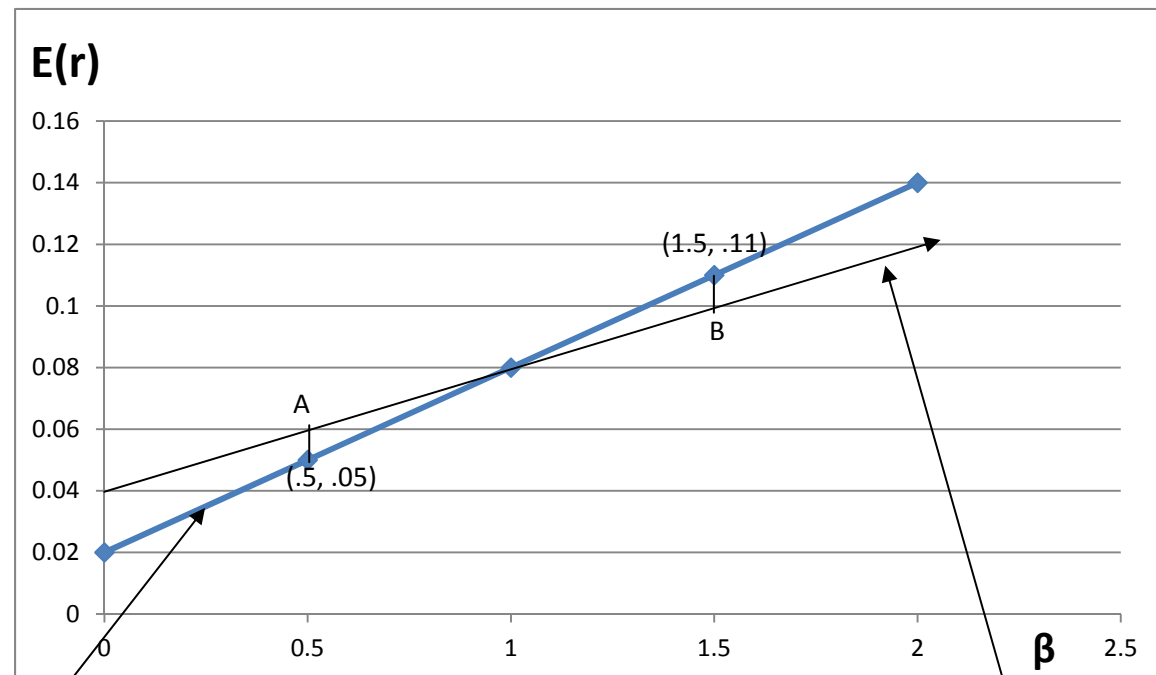
SML implied by expected Mkt return

Slope $E(r_M) - r_f = .08 - .02 = .06$

Empirically estimated

SML: slope $= \dfrac{.1 - .06}{1.5 - .05} = .04$

$E(r_A) = .02 + .5(.08 - .02) = .05$

$E(r_B) = .02 + 1.5(.08 - .02) = .11$

Sample 2:

E(r) %

1) implied SML by expected market return

12

10

B

8

A

6

2) empirically estimated SML

4

2

0

0   0.2   0.4   0.6   0.8   1   1.2 **β** 1.4   1.6
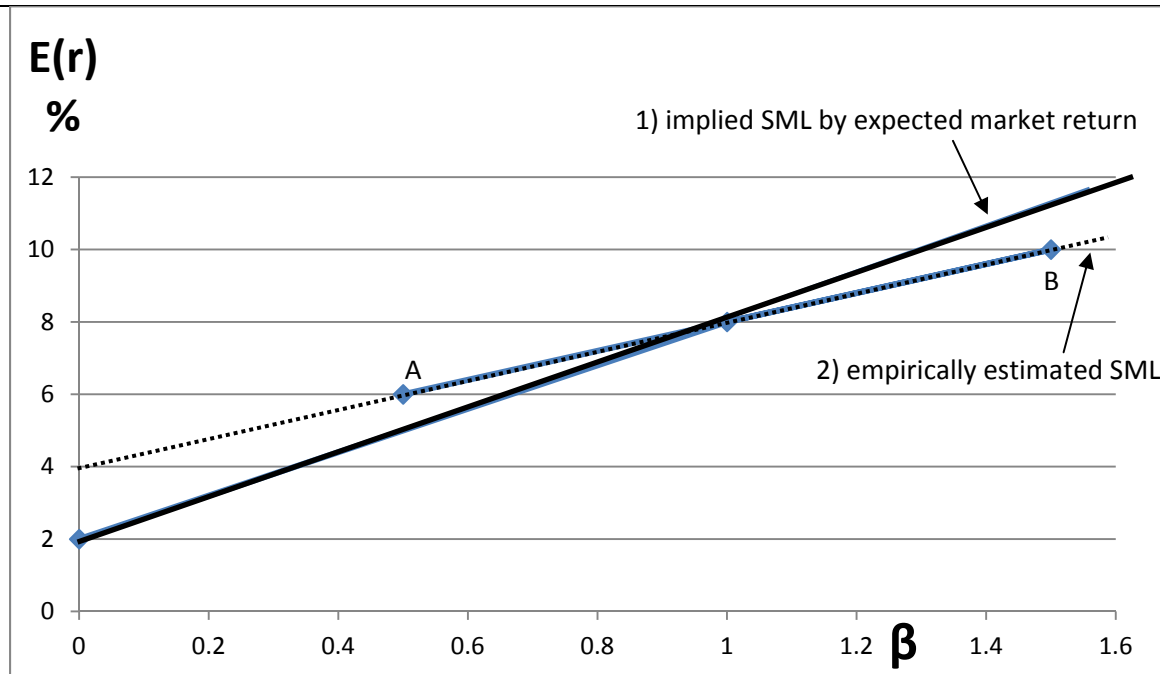
1) SML = .02 + .06β
2) SML = .04 + .04β

Sample 3:

Based on exp market returns the SML will have slope .08 -.02 = .06 $E[R_i] = \beta(.06)$

Based on empirical $r_i - r_f = \gamma_0 + \beta \gamma_1$

   A $.04 = \gamma_0 + .5 \gamma_1$ $.04 + .5(.038)$
   B $.08 = \gamma_0 + 1.5 \gamma_1$

Solve for γ_0 and γ_1 =>

            $.04 = 1 \gamma_1$ $\gamma_1 = .04$
            $.04 = \gamma_0 + .5(.04)$ => $\gamma_0 = .02$

                        $E[R_i] = .02 + \beta(.04)$

Expected risk premium $E[r_i] - r_f$

SML from expected market (intercept = 0, slope .06)

B (1.5, .08)

SML from empirical intercept .02, slope .04

A (.5, .04)

Beta
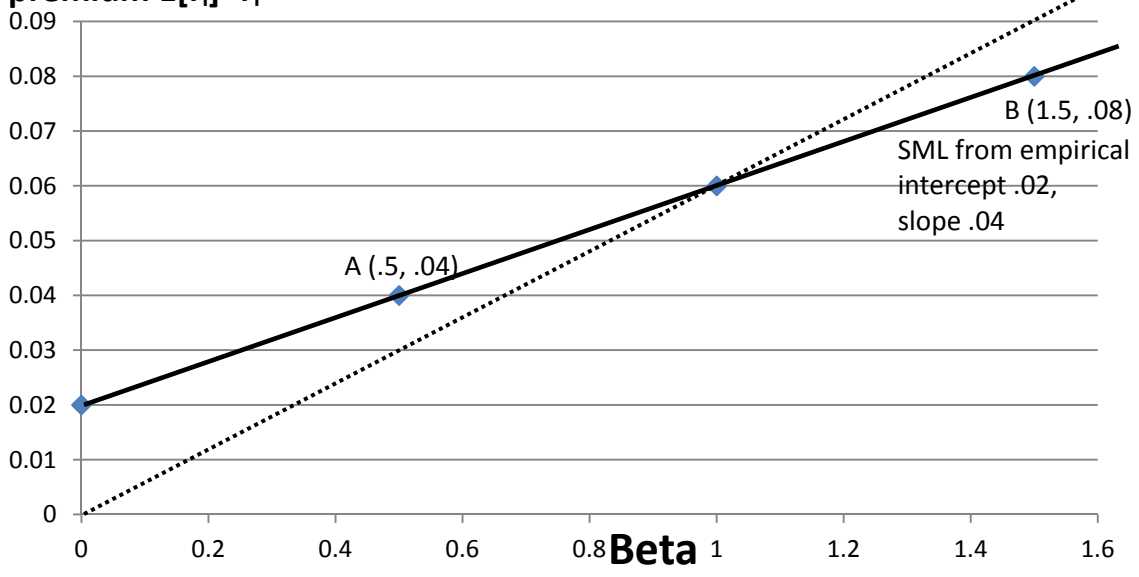
Part b: *0.5* point(s)

Part b: 0.5 point

Sample 1:

Stock A has a positive α, $α_A$ = 1% = 6% -[2% + 0.5 (8% -2%)]
Stock B has a negative α, $α_B$ = -1% = 10% -[2% + 1.5(8% -2%)]

Therefore, Stock A is a better buy based on CAPM.

Sample 2:

A is a better buy because the expected return is higher than the expected return of the same risk in the market (0.05) estimated by CAPM

B is lower than the expected return than the CAPM suggested so not a good buy

Sample 3:

Stock A is a better buy according to CAPM
Since it is above the SML (B is below the SML)

EXAMINER'S REPORT

General Commentary

- Candidates expected to know how to graph the SML implied by the expected market return (line with intercept at risk free rate and going through market rate); also be able to graph empirical SML going through points A and B; be able to assess how stocks are performing with respect to CAPM expectations
- Candidates generally scored very well – biggest difficulty was in graphing the empirical SML (just under half got credit for this)
  - Many candidates correctly graphed A & B but did not draw a line through the two points for the empirical SML; many also incorrectly graphed the line - for example, drawing the line through A & B but then having it intersect the Y axis at the risk free rate. Some people also drew two empirical lines – one through the risk free rate at the Y axis and point A and one through the risk free rate at the Y axis and point B. Some also drew an empirical SML that did not go through A & B at all.
- Most errors were related to candidates overlooking specific directions or mental errors, such as forgetting to draw the empirical SML, reversing A & B on the graph, so shifting the entire graph down by the risk free rate but not labelling as such.

Part a

- Candidates expected to know how to graph the SML implied by the expected market return (line with intercept at risk free rate and going through market rate); also be able to graph empirical SML going through points A and B;
- To receive full credit, candidate should have graphed and labelled both the implied SML and the empirical SML, graph and label points A & B, and label the axes
- Common errors – not graphing the empirical SML, not labelling something (lines, points or axes)
- For the labelling, we accepted both giving the point coordinates for A & B or labeling the numbers on the axis so that it was clear what the A & B coordinates were

Part b

- Candidates expected to be able to assess how stocks are performing with respect to CAPM expectations
- To receive full credit, candidate was expected to identify which stock is a better buy and explain why
- Common error – using positive alpha as the explanation but not identifying what alpha is; candidates could receive full credit without mentioning alpha (eg explaining that A is better because it lies above the implied SML), but if positive alpha was used as the only explanation, it needed to be defined. We accepted both explicit calculation of alpha and also graphing and labelling it as definition.

QUESTION *4*	
TOTAL POINT VALUE: *1.5*	**LEARNING OBJECTIVE:** *Part A, LO 5*
SAMPLE ANSWERS	
Part a: *1* point	

Sample 1:

a) The number of estimates required to run the Markowitz model on 200 stocks is enormous. Not only would you need expected return and variance estimates for each stock, but also covariances between every stock. This is extremely labor intensive.

   Then with so many estimates, it is likely that some may not actually produce sensical results. The covariances especially may not make sense when all the data is rolled up.

b) The number of estimates required in a single index model is significantly fewer. Covariances are calculated using variance of the market index and the β_is of each security.

   With so many fewer estimates it seems mutually inconsistent results are far less likely. However, sources of correlation outside the market, like perhaps related to type of firm or geographical location are missed.

Sample 2:

a) 1. Many parameter estimates are required. For each stock we need an estimate of expected return and variance. We also need estimates of covariances for every pair of stocks, not just amongst the 200 tech stocks, but all in portfolio. Making this many estimates leads to high risk of parameter error in our model.

   2. Since we need to estimate covariances between each of these tech stocks and all other stocks in our portfolio, our analysts cannot specialize in the technology sector. They need to know other sectors to estimate the covariance of a tech stock with an oil stock for example.

b) 1. Single index model overcomes this issue by substantially reducing the number of parameter estimates, *from* $\frac{(n^2 + 3n)}{2}$ to 3n + 2.

   2. Single index also overcomes this issue as covariance is determined by the securities' covariance with the index. Cov(A,B) = $\beta_A \beta_B \sigma_M^2$, so specialization is possible.

Sample 3:

a) Markowitz model need to estimate a lot of parameters for the portfolio =
   -expected returns for each stock
   -std. dev. for each stock
   -covariance among the 200 stocks
   -and market expected return and variance
   Due to the large numbers of parameters to be estimated, there is likely prediction error in the model.

b) Single index model significantly reduced the number of parameters to be estimated. Due to reduced estimates, it improves the models efficiency and accuracy.

Sample 4:

a) 1. There are too many parameters. Need to calc correlation & covariances between every single pair of stocks, resulting in intensive computation.

2. Doesn't separate sys risk & firm-specific risk when calc'ing covariances and variances. Hence not clear & easy to understand sources of risks.

b) 1. Single-index has less inputs, where it separate sys & firm-specific differences. All stocks are correlated with one common sys risk factor, hence a lot less parameters to work with.

2. Single-index quantifies sys & firm-spec risks separately. Hence the model is more clear where the risks come from. Easier to interpret.

Sample 5:

a) i) Need a full covariance matrix for covariance between each pair of assets
      - this is very computationally inefficient
  ii) You would also want to gather historical data for the securities, which may be difficult for certain technology companies because they haven't been around that long

b) i) Single-index model assumes covariance is driven by a common factor, so this greatly reduces inefficiency from part a) above
ii) Single-index model still needs historical data just like Markowitz, so no change with this issue from part a) above

EXAMINER'S REPORT

General Commentary

This question tested Learning Objective 5 from Part A of the syllabus, asking candidates to compare issues between the Markowitz model and the single index model. Candidates generally scored well with over half of the candidates scoring full credit.

Part a

In Part a of this question, the graders expected the candidates to identify and briefly discuss two issues with the Markowitz model.
One common mistake that candidates made was not giving enough information about an issue. For example, a response that simply said, "The Markowitz model needs a lot of parameters to be estimated" would not get as much credit as a response that said, "The Markowitz model needs a lot of parameters to be estimated, driven primarily by the need for the covariance matrix to be estimated between every pair of stocks in the portfolio."
Another mistake some candidates made was identifying an issue that was not of particular concern to the Markowitz model and not giving supporting detail that convinced the graders it was a concern for the Markowitz model. For example, if a candidate simply said, "The Markowitz model experiences

model risk" this was not deemed specific enough to Markowitz as this statement is obviously true for any model.

Part b

For Part b of the question, the graders expected the candidates to identify whether the single index model overcomes the issues identified in Part a, with a bit of supporting information. Any answer here could receive credit if the argument was sound. For example, if a candidate had identified 'estimation error' as an issue with the Markowitz model, he could either argue that the single index model overcomes this issue because it has fewer parameter estimates so it is exposed to less estimation error, or he could argue that the single index model does not overcome this issue because it is overly simplified.

Common mistakes were not responding to the issues identified in Part a (i.e., listing random facts about single index models that didn't pertain to whether the model overcame the issues identified for Markowitz) or providing faulty rationale for an argument. For example, some candidates responded single index model overcomes parameter risk because its less risky. This is not a sufficient reason. Stating the single index model overcomes parameter risk due to fewer parameter estimates would be an acceptable answer.

QUESTION *5*	
TOTAL POINT VALUE: *2.75*	**LEARNING OBJECTIVE: *A1, A2, Ad***
SAMPLE ANSWERS	

Sample 1:

$$W_A = \frac{E(R_A)\sigma_B^2 - E(R_B)Cov(AB)}{E(R_A)\sigma_B^2 + E(R_B)\sigma_A^2 - [E\ (R_A) + E(R_B)]Cov(AB)}$$

$E(R) = E(r) - r_f$, $Cov(AB) = \rho_{AB}\ \sigma_A\ \sigma_B$

$$W_A = \frac{.06(.28^2) - .08(-.4)(.2)(.25)}{.06(.28^2) + .08(.2^2) - (.06 + .08)(-.4)(.2)(.25)} = .5487$$

$W_B = 1 - W_A = = .4513$

$E(r_p) = W_A E(r_A) + W_B E(r_B) = .5487(.08) + .4513(.1) = .089$
$\sigma^2_p = W_A^2\ \sigma^2_A + W_B^2\ \sigma^2_B + 2W_A W_B Cov(AB) = .5487^2(.2^2) + .4513^2(.25^2) + 2(.5987)(.4513)(-.4)(.2)(.25)$
    $= .01487$

If $u = E(r_c) - \frac{1}{2}A\ \sigma^2_c$ $y^* = \frac{E(r_P) - r_f}{A\ \sigma^2_p} = \frac{.089 - .02}{5(.01487)} = .9282$

$E(r_c) = y^*E(r_p) + (1-y^*)r_f = .9282(.089) + (1-.9282)(.02) = .084$
$\sigma_c = y^*\ \sigma_p = .9282\ \sqrt{.01487} = .1132$

Sample 2:
Rf = 2%
Cor (A,B) = -.04
A = 5

1. First derive the optimal risky portfolio
covariance of A, B is
Cov(A,B) = - correlation * σ_A * σ_B
        = -0.4 x 0.2 x 0.25 = -0.02
Proportion of asset A in the optimal risky portfolio
$W^*_A = \frac{E(R_A)\ \sigma_B^2 - E(R_B)\ Cov(A,B)}{E(R_A)\ \sigma_B^2 + E(R_B)\ \sigma_A^2 - [E(R_A) + E(R_B)]\ Cov(A,B)}$

$= \frac{(8\% - 2\%)*25\%^2 - (10\% - 2\%)*(-0.02)}{6\% \times 25\%^2 + 8\% * 20\%^2 - (6\% + 8\%) * (-0.02)}$

$= \frac{0.00535}{0.00975} = 54.87\%$

Proportion of asset B $W_B^* = 1 - 54.87\% = 45.13\%$

$E(r_P) = W_A^* E(r_A) + W_B^* E(R_B)$
    = 54.87% * 8% + 45.13% x 10% = 8.90%

$\sigma_p^2 = W_A*^2\,\sigma_A^2 + W_B*^2\,\sigma_B^2 + 2W_A*W_B*Cov(A,B)$

$\quad = 54.87\%^2 \times 0.2^2 + 45.13\%^2 \times 0.25^2 + 2 \times 54.87\% \times 45.13\% *(-0.02)$

$\quad = 0.014867$

2. Assume Utility $U = E(r) - \dfrac{1}{2}\,A\,\sigma^2$

proportion is risky assets $\quad y* = \dfrac{E(r_p) - r_f}{A\,\sigma_p^2} = \dfrac{8.9\% - 2\%}{5 \times 0.014867}$

$$= 92.86\%$$

3. Expected return of optimal complete portfolio

$E(r_c) = y*E(r_p) + (1 - y*) - r_f$

$\quad = 92.86\% \times 8.90\% + (1 - 92.86\%) \times 2\%$

$\quad = 8.41\%$

s.d. $\sigma_c = y* \,\sigma_p$

$\quad = 92.86\% * \sqrt{0.014867}$

$\quad = 92.86\% \times 0.12193 = 0.1132$

EXAMINER'S REPORT
The question followed the methodology outlined in the text. Candidates scored really well on this question with most getting nearly full credit. The pieces where most tended to lose credit were simple formula errors (plus instead of a minus sign, forgetting to multiply by 2.0, confusing A with B, etc.)

QUESTION 6	
TOTAL POINT VALUE: 2.0	**LEARNING OBJECTIVE: A8, A9**
SAMPLE ANSWERS	

Part a: 0.5 point(s)

Sample 1:

-Small firms tend to outperform big firms as shown by $E(r_{small})$ = 9% > $E(r_{big})$ = 5%

   -High B/M firms tend to outperform low B/M firms,

   $E(R_H)$ = 7% > $E(r_L)$ = 4%

   -Semistrong EMH states that stock price reflects all publicly available information. Since it is easy to sort firms by size and B/M ratio, this seems to violate semistrong EMH.

Sample 2:

1. Small capitalization stocks outperform big capitalization stocks.
2. High book to market ratio stocks out perform low book to market ratio stocks.

Part b: 1.0 point(s)

Sample 1:

Fama-French Model

$E(R_i) = \alpha_i + \beta_i\, E(R_m) + \beta_{SMB}E(SMB) + \beta_{HML}E(HML)$

$E(R_A)$ = 2% + 1.0 (6%-1%) + 0.8 (9%-5%) + 1.1 (7% - 4%)

    = 0.135

$E(r_A)$ = 0.135 + 1% = 14.5%

Sample 2:

$E[r_A] = r_f + \beta_m R_m + \beta_{SMB}(R_S - R_B) + \beta_{HML}(R_H\text{-}R_L)$

   = .01 + (1)(.06 - .01) + (.8)(.09-.05) + (1.1) (.07-.04)

$E[r_A]$ = .125

> No firm specific influences

Part c: 0.5 point(s)

Sample 1:

Liquidity is not a priced factor in the Fama-French three-factor model. Therefore stocks with liquidity risk will have a higher α value than a similar stock without liquidity risk.

Sample 2:

Investors prefer liquidity; they will require a premium for illiquidity in the form of α.

Sample 3:

Fama-French α are high for high liquidity beta portfolios and low for low liquidity beta portfolio.

EXAMINER'S REPORT

Candidates were expected to be familiar with the semi-strong form of the efficient market hypothesis (part a), and have an understanding of the Fama-French three-factor model (parts b,c).

Overall, candidates scored well on this question. Part c seemed to be the most challenging (see explanation below).

Part a
Candidates were expected to know the kinds of market anomalies that would contradict the semi-strong form of the efficient market hypothesis, and which of these anomalies were present in the portfolio returns provided. A full credit response would have mentioned that expected returns varied by market capitalization and book-to-market ratio. In general, candidates performed very well on part a. The most common error was to include a contradiction of the semi-strong form of the EMH that was not ascertainable from the portfolio of stocks provided (ex. post-earnings announcement drift)

Part b
Candidates were expected to know the Fama-French three-factor model, and to calculate an expected return when all of the model inputs were provided. Responses that received full credit would have correctly set up the Fama-French model using the information provided in the question (including calculation of the market risk premium, the Small Minus Big factor, and the High Minus Low factor) and correctly calculated the expected rate of return. Overall, candidates had some difficulty attaining full credit for part b. The most common error was a failure to recognize that the Fama-French model results in an expected risk premium, and that the risk free rate should be added to this result in order to arrive at the expected return. In addition, some candidates took the average of the small, medium, and big stock returns when determining the Small Minus Big factor, instead of taking the small stock return minus the big stock return. This same error was seen for the calculation of the High Minus Low factor.

Part c
Candidates were expected to know the components of the Fama-French three-factor model, and also have an understanding of how liquidity would impact expected return. A full credit response would have mentioned that lower liquidity would result in a higher expected return (or higher liquidity and a lower expected return) and that the liquidity impact would be contained within the α component. Full credit was also given for responses which stated that the Fama French model does not explicitly consider liquidity, but any impact would be contained within the α component. Overall, most candidates had difficulty attaining full credit for part c. The most common error was to state that the Small Minus Big factor and High Minus Low factor served as proxies for liquidity exposure within the Fama-French model.

QUESTION *7*	
TOTAL POINT VALUE: *1.75*	**LEARNING OBJECTIVE: B2**
SAMPLE ANSWERS	

The problem can be logically split into two parts. Part 1 - Calculating the Forward Rate from t=2 to t=3, then Part 2 - calculating the value of the Forward Rate Agreement

Calculate the rates from the Zero Coupon Bonds:

Bond 1: $r_1 = (100 / 95.12) - 1 = .0513$ or Continuous: $r_1 = \ln(100/95.12) = .050031$

Bond 3: $r_3 = (100 / 78.66)^{1/3} - 1 = .0833$ or Continuous: $r_3 = \ln(100/78.66) / 3 = .080012$

Using Bond 4, set up an equation to find R2.

$91.72 = (5 / 1.0513) + (5 / (1 + r_2)^2) + (105 / 1.0833^3)$

$=> r_2 = .0695$

Or it could be worked directly as:

$91.72 = (5 * .9512) + (5 * D2) + (105 * .7866)$

Where D2 = .8742, or inversed is $(1+r_2)^2 = 1.1439$ or $r_2 = .0695$

or Continuous: $91.72 = (5 / e^{.05}) + (5 / e^{r2*2}) + (105 / e^{.08 * 3})$

$r_2 = .067575$

Find the Forward rate at t=3 or: $f_3 = [(1 + r_3)^3 / (1+ r_2)^2] - 1$

⇨ $f_3 = [(1.0833)^3 / (1.0695)^2] - 1 = .111436$ or 11.14%

⇨ $f_3 = (.8742 / .7866) - 1 = .1113654$ or 11.14%

⇨ $f_3 = [\exp^{(.08 * 3)} / \exp^{(.06757 * 2)}] - 1 = .110555$ or 11.06%

- Used inputs $r_1=1.0513$, $r_3=1.0833$ => $r_2=1.0695$; $f_3=1.111436$ or 11.14%
- User inputs $r_1=1.051$, $r_3=1.083$ => $r_2=1.078$; $f_3=1.09307$ or 9.31%
- User Inputs Continuous: $r_1=.0500$; $r_3=.0800$; $r_2=.0675746$; $f_3=.110555$ or 11.06%; depending on how one rounds r_2, f_3 could round to 11.05% or 11.07%.
- *……}*

Find the Forward Rate Agreement value to Company B:

$V_{FRA} = L * (R_K - R_F) * (T_2 - T_1) * \exp^{-R2*T2}$

$V_{FRA} = \$8,000,000 * (f_3 - .1200) * (3 - 2) *$ Discount Factor

Due to differences in rounding in the first part of the problem, different numerical valid answers would result. There are also three different Discount Factors that were commonly used.

Discount Factor $(1.0833)^{-3}$ or 0.7866 are roughly equivalent. The third variation would be to discount by the continuous rate as in discount = $\exp^{-(.08 * 3)}$

A few candidates discounted by the forward rate (1.1114) and then discounted by $(1.0695)^2$ which is equivalent to $(1.0833)^3$

- $f_3 = .1114$; $DF = (1.0833)^{-3}$ or $0.7866 \Rightarrow \$-54{,}118$
- $f_3 = .1114$; $DF = \exp^{(-.08 * 3)} \Rightarrow \$-54{,}120$
- $f_3 = .111$; $DF = (1.0833)^{-3}$ or $0.7866 \Rightarrow \$-56{,}635$
- $f_3 = .111$; $DF = \exp^{(-.08 * 3)} \Rightarrow \$-56{,}637$
- $f_3 = .111436$; $DF = (1.0833)^{-3}$ or $0.7866 \Rightarrow \$-53{,}892$
- $f_3 = .111436$; $DF = \exp^{(-.08 * 3)} \Rightarrow \$-53{,}893$
- $f_3 = .11143649$; $DF = (1.0833)^{-3}$ or $0.7866 \Rightarrow \$-53{,}888$
- $f_3 = .11143649$; $DF = \exp^{(-.08 * 3)} \Rightarrow \$-53{,}890$
- $f_3 = .11144$; $DF = (1.0833)^{-3} \Rightarrow \$-53{,}866$
- $f_3 = .11137$; $DF = (1.0833)^{-3}$ or $0.7866 \Rightarrow \$-54{,}307$
- $f_3 = .11136537$; $DF = (1.0833)^{-3}$ or $0.7866 \Rightarrow \$-54{,}336$
- $f_3 = .11136537$; $DF = \exp^{(-.08 * 3)} \Rightarrow \$-54{,}338$
- $f_3 = .111361$; $DF = \exp^{(-.08001 * 3)} \Rightarrow \$-54{,}364$
- $f_3 = .11147$; $DF = 1.2713^{-1} \Rightarrow \$-53{,}674$
- $f_3 = .1115$; $DF = (1.0833)^{-3} \Rightarrow \$-53{,}489$
- $f_3 = .1115$; $DF = \exp^{(-.08 * 3)} \Rightarrow \$-53{,}491$
- continuous: $f_3 = .1106$; $DF = \exp^{(-.08 * 3)} \Rightarrow \$-59{,}154$
- continuous: $f_3 = .1105448$; $DF = \exp^{(-.08 * 3)} \Rightarrow \$-59{,}501$
- continuous: $f_3 = .11054$; $DF = \exp^{(-.08 * 3)} \Rightarrow \$-59{,}532$
- continuous: $f_3 = .110555$; $DF = \exp^{(-.08 * 3)} \Rightarrow \$-59{,}437$
- continuous: $f_3 = .1105$; $DF = \exp^{(-.08 * 3)} \Rightarrow \$-59{,}784$
- continuous: $f_3 = .1107$; $DF = \exp^{(-.08 * 3)} \Rightarrow \$-58{,}525$

- $f_3 = .0931$; $DF = (1.083)^{-3}$ or $.7866 \Rightarrow \$-169{,}417$ or $\$-169{,}276$
- $f_3 = .0931$; $DF = \exp^{(-.08 * 3)} \Rightarrow \$-169{,}282$
- $f_3 = .093$; $DF = (1.083)^{-3}$ or $(.7866) \Rightarrow \$-170{,}047$ or $\$-169{,}906$
- $f_3 = .093$; $DF = \exp^{(-.08 * 3)} \Rightarrow \$-169{,}912$

An alternative method after calculating D2 = .8742:
$V_{FRA} = \$8M * (.8742) - \$8M * (1.12) * .7866 = \$-54{,}336$

A similar alternative method without calculating f3:
$V_{FRA} = \$8M / (1.0695)^2 - \$8M * (1.12) / (1.0833)^3 = \$-54{,}336$

EXAMINER'S REPORT

- The candidate was expected to know how to get the Forward Rate at t=3 using the Bootstrapping Method, and then use that information to Calculate the value of the Forward Rate Agreement.
- Candidates generally received full credit, with the exception of minor deductions for calculating errors, or expressing the FRA in terms of Company A, rather than Company B.
- Some candidates did not discount by a correct rate, or discounted for two years rather than three. Some used $\exp^{(.0833 * 3)}$ where .0833 is not a continuous rate in the problem given.
- A few candidates misinterpreted the listed Bond 4 as a 4-year Bond, rather than a 3-year

Bond, then could not solve properly for r_2.

- A few candidates calculated f_2 rather than f_3 and used f_2 in the FRA equation.

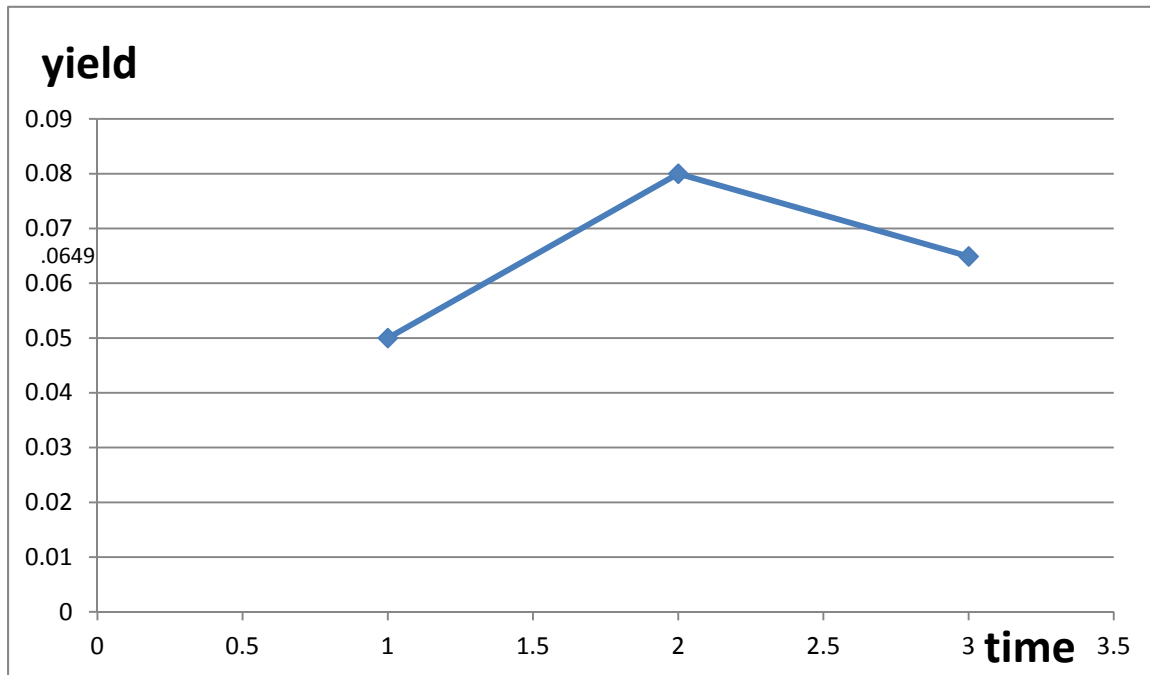| QUESTION 8	
TOTAL POINT VALUE: *2.0*	LEARNING OBJECTIVE: *B1, B2*
ANSWERS	
Part a: *1.5* point(s)	

Sample 1:

Bond A price = Face value $(1+r_1)^{-1}$

$951.23 = 1000 \, e^{-r_1}$

$r_1 = 5\%$

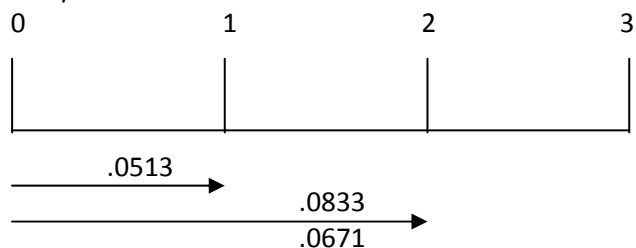Assuming all rates should be continuously compounded as that is how 2 yr spot rate is provided

$r_2 = 8\%$

Bond C $901.78 = 30e^{-r_1} + 30e^{-r_2 x 2} + 1030e^{-r_3 x 3}$

.03 x 1000

$3r_3 = .1948$

$r_3 = 6.49\%$

a)

yield

b) The yield curve above reflects the expectations theory of term-structure. Rates are expected to rise for 2 yrs, but then decrease after that.

Sample 2:

0   1   2   3

.0513

.0833

.0671

Assume all interest rates are annual unless

specified otherwise

$951.23 = \frac{1000}{1+y_1}$ $y_1 = .0513$

$E^{-.08(2)} = \frac{1}{(1+y_2)^2}$ $y_2 = .0833$

$901.78 = \frac{30}{1.0513} + \frac{30}{1.0833^2} + \frac{1030}{(1+y_3)^3}$ $y_3 = .0671$
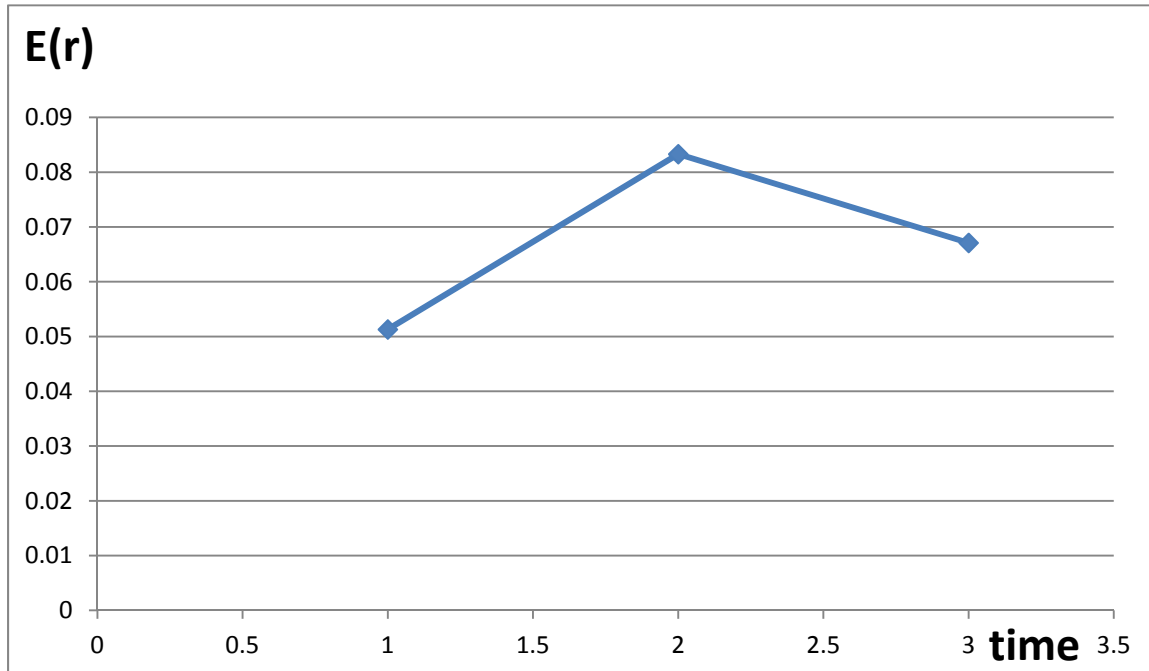
b. Segmentation theory – the yields offered at each point in time (short, medium, long term) are driven by the supply and demand for bonds at that time.

There must be a low demand for bonds at t=2, which is why it is offering a higher rate than at times 1 and 3.

EXAMINER'S REPORT

General Commentary

Most candidates scored well. Average scores were around 1.6 out of 2.0 points. 37% of the candidates got a perfect score, while another 20% scored 1.75 out of 2.0 points.

Part a

Candidate should be able to calculate the rates at 3 different maturities, and present it clearly with a graph, in order to obtain full credit.

Common errors were due to miscalculation. Other candidates confused forward rates with zero rates.

Part b
Candidates should be able to explain the different term structure theories to obtain full credit.
Most candidates got this part correctly. One of the common mistakes here is the demand supply relationship and how it affects the yields. A number of candidates incorrectly thought that a higher demand would correspond to a higher yield.

QUESTION 9 – Exam 9 Spring 2014	
TOTAL POINT VALUE: 2.5 points	**LEARNING OBJECTIVE: B.3 – Utilize various strategies to manage interest rate risk and cash flow risk in a bond portfolio**

SAMPLE ANSWERS

Part a: 0.5 point

- **Sample answer #1**

For company A & B, the difference in floating is (LIBOR – 0.2%) – (LIBOR + 0.9%) = -1.1%. In fixed: 4.3% – 5.4% = -1.1%

The comparative advantage occurs when one company pays relatively less than the other in one market versus the other. Here company B pays 1.1% more in both markets.
Total advantage = 1.1% – 1.1% = 0%.

- **Sample answer #2**

Comp. Adv. Gain (CAG) = (Dif in float rates) – (Dif in fixed rates)
= (LIBOR + .9% – (LIBOR – .2%)) – (5.4% – 4.3%)
= 1.1% – 1.1% = 0%

No comparative advantage gain between A, B lending rates, so swap wouldn't provide comp. adv. benefit b/c the dif. in lending rates is same between A, B in both markets.

- **Sample answer #3**

Δ in floating rate = 0.9% - (-0.2%) = 1.1%
Δ in fixed rate = 5.4% - 4.3% = 1,1%

Since the differences in floating/fixed rate are the same, (1.1%), no company has comparative advantage in each market. So they have to go with the rates offered. No benefits could be achieved.

- **Sample answer #4**

A comparative advantage exists when difference between the floating rate is larger than the difference between the fixed rates (or vice versa) between the 2 companies. Since the rate differences are the same no comparative advantage exists.

    (LIBOR – 0.2%) – (LIBOR + .9%) = (-1.1%) = 4.3% - 5.4%

Part b: 2 points

- **Sample answer #1**

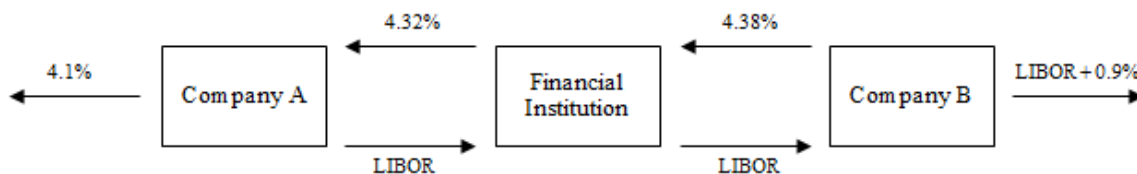Floating: (LIBOR – 0.1%) – (LIBOR + 0.9%) = -1.0%.
Fixed: 4.1% – 5.4% = -1.3%
Entire advantage to be split between companies A, B, and the financial intermediary is 1.3% – 1% = 0.3%.
Company A has comparative advantage in fixed market and company B has advantage in floating market. These are the markets where each must initially establish their loans.
The 0.3% split – 0.06% to financial intermediary leaves a gain of 0.24% for the two companies. Assume they split it evenly for 0.24% / 2 each = 0.12%
Company A's final rate should be floating LIBOR – 0.1% – 0.12% = LIBOR – 0.22%

```
                        4.32%                    4.38%
          4.1%     <---------            <---------            LIBOR+0.9%
     <-------  [Company A]      [Financial      [Company B]  --------->
                               Institution]
                LIBOR  --------->          LIBOR  --------->
```

- **Sample answer #2**

CAG = -[(LIBOR + .9% – (L – .1%)) – (5.4% – 4.1%)]
=-[1% – 1.3%] = 0.3%

Split CAG
A .12%
B .12%
F.I .06%

Final Swap:
A Pays LIBOR
A Receives 4.32%
B Pays 4.38%
B Receives LIBOR
F.I. Gains 0.06%

Diagram (top):

Fixed Loan — 4.1% ← Company A — 4.1% → Financial Institution — 5.4% - .12% → Floating — LIBOR+0.9% → Company B
50M → Company A → LIBOR - .1% - .12% → Financial Institution → LIBOR+0.9% → Company B
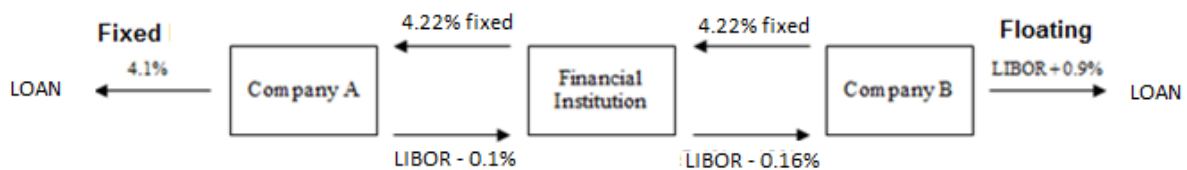Company B → 50M
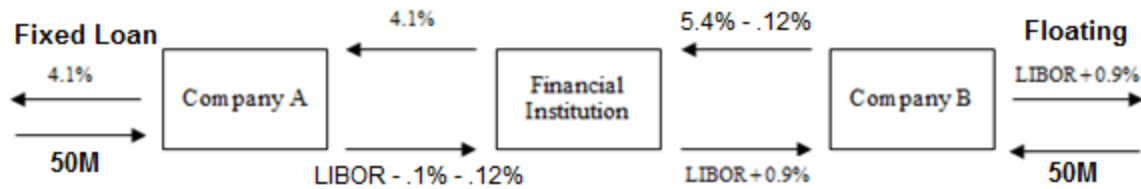
- **Sample answer #3**

Comparative Advantage
   Floating: 1.0
   Fixed: 1.3

→ Company A has comparative advantage in fixed & B has comparative advantage in floating
→ Comparative advantage = 0.30 − 0.06 = 0.24% pts → 0.12% for each of A & B

Diagram (bottom):

Fixed — 4.1% ← Company A — 4.22% fixed → Financial Institution — 4.22% fixed → Floating — LIBOR+0.9% → Company B
LOAN ← Company A — LIBOR - 0.1% → Financial Institution — LIBOR - 0.16% → Company B → LOAN

A: pays LIBOR − 0.1% to bank
   Pays 4.1% to fixed loan
   Receives 4.22% fixed from bank
     → net gain = 0.12%

B: pays LIBOR + 0.9% to loan
   Pays 4.22% fixed to bank
   Receives LIBOR − 0.16% from bank
     → net gain = LIBOR + .9 + 4.22 − (LIBOR - .16) − 5.4 = 0.12%

I did not adjust floating part to LIBOR as this is optional.

EXAMINER'S REPORT

- The candidate was expected to know what "comparative advantage" means in terms of the difference between floating and fixed interest rates for two companies. Given their knowledge of what comparative advantage was, they were also expected to be able to design an interest rate swap, between the two companies, and the financial institution.
- Generally candidates did very well on this question, and the majority got full credit, or very

close to full credit on this question.
- o The only area with some difficulty was in determining the middle interest rates in the swap, that went from Company A, to the financial institution, to Company B, and back the other way. This was the most difficult portion, however most candidates were able to answer this correctly, or the majority of it correctly.

Part a

- Candidates were expected to be able to explain why the first set of interest rates would not result in a comparative advantage swap.
- To receive full credit for part a, candidates were expected to be able to know or identify what the comparative advantage was for the first set of interest rates, and then be able to explain why these interest rates between the two companies do not result in a comparative advantage.
- Common errors included the candidates not being able to fully explain that the difference had to be more than or less than one another in either the fixed or floating. Most people were able to obtain full credit, but those that couldn't often just used the same words, "comparative advantage" or a slight modification, for example "competitive advantage" to define what was meant.

Part b

- Candidates were expected to be able to design an interest rate swap, where both company A and company B would receive better rates, in either the fixed or floating interest rate markets, once they completed the swap, and that the financial institution would gain .06%. They were expected to be able to identify that Company A has a comparative advantage in the fixed market, and that Company B has a comparative advantage in the floating market. Candidates did not have to have both companies benefit by an equal amount, but almost every candidate did.
- To receive full credit candidates needed to be able to identify:
  - o What the comparative advantage is between these two companies, and thus how much additional gain there was to split between the two companies, once the financial institution received .06%
  - o Recognition that Company A has comparative advantage in fixed market, and Company B has comparative advantage in floating market, and thus which interest rate each of Company A and Company B will borrow at
  - o The candidate also needed to be able to identify the interest rate flows between Company A to the financial institution to Company B, and the flows in the opposite direction from Company A to the financial institution to Company B
- Majority of candidates received full credit on part b, though not quite as many as those who received full credit on part a. Common errors on part b included:
  - o Usually the mistakes were made for the interest rate flows between the financial institution and each company
  - o A few candidates incorrectly stated Company B would borrow initially in the fixed market and Company A in the floating market, but this was relatively few candidates
  - o There were also a few calculation errors in not getting the right comparative

advantage, thus the interest rate flows would not work out exactly right, however this also was fairly rare

QUESTION *10*	
TOTAL POINT VALUE: *2*	**LEARNING OBJECTIVE: *B6***
SAMPLE ANSWERS	

Part a: 0.5 point

- **Sample answer #1**

$$F = \frac{S(k-y)}{1+y} \times \frac{d}{1-d} \quad \text{where } d = \frac{cr}{1+y}$$

$$d = (.88)/1.05 = .8381$$

$$F = \frac{65m(.12-.05)}{.1.05} \times \frac{.8381}{.1619} = 22{,}432{,}159.8$$

- **Sample answer #2**

From Panning's Formula, with k = a+by model

$$F = \frac{crs(a+by-y)}{(1+y)(1+y-cr)}$$

$$= \frac{0.88(65)(0.12+0(0.05)-0.05)}{(1+0.05)(1+0.05-0.88)}$$

$$= 4.004/0.1785 = 22.4314$$

- **Sample answer #3**

S − 65M   k = 0.12   a = 0.12
  cr = 0.88   y = 0.05   b = 0

$$P = \frac{S(k-y)+L}{1+y} + E$$

$$P - E - \left(\frac{L}{1+y}\right) = \frac{65M(0.12-0.05)}{1.05} = 4.33M \qquad\qquad d = \frac{cr}{1+y} = 0.8381$$

$$F = \left(P - E - \frac{L}{1+y}\right)\frac{d}{1-d} = 4.33M \times \frac{0.8381}{1-0.8381} = 22.431M$$
$$= 22\,431\,373$$

Part b: 0.5 point

- **Sample answer #1**

$$D = \frac{a-b+1}{(k-y)(1+y)} + \frac{1}{1+y-cr}$$

$$= \frac{.12-0+1}{(.12-.05)(1.05)} + \frac{1}{1.05-.88} = 21.12$$

- **Sample answer #2**

From Panning's formula

$$D_F = \frac{a-b+1}{(1+y)(a+by-y)} + \frac{1}{1+y-cr}$$

$$= \frac{0.12-0+1}{(1+0.05)(0.12+0(0.05)-0.05)} + \frac{1}{1+0.05-0.88}$$

$$= \frac{1.12}{0.0735} + \frac{1}{0.17}$$

$$= 21.12$$

- **Sample answer #3**

$$D_F = \frac{a-b+1}{(1+y)(a+by-y)} + \frac{1}{1+y-cr} = \frac{0.12+1}{(1.05)(0.12-0.05)} + \frac{1}{1.05-0.88}$$
$$= 21.12$$

Part c: 1 point

- **Sample answer #1**

Strategy 2 has the higher Franchise Value so I would recommend choosing that strategy. It does have quite a bit higher duration making it susceptible to int rate risk but I would still choose it. If interest rates start to move, the insurer can adjust the pricing strategy – Maybe choose a + b parameters in between the 2 options.

- **Sample answer #2**

Recommend strategy 1
  - similar Franchise value as original (22430 vs 22431.4)
      - doesn't impact company performance
  - much lower duration as original (6.83 vs 21.12)
  - lower duration than 2
      - easier to find asset for duration matching

- **Sample answer #3**

Pursue with strategy 1. It may have a slightly lower franchise value than strategy 2 but the duration is much lower, making the franchise value much less impacted by changes in interest rate.

EXAMINER'S REPORT

- The candidate was expected to be familiar with franchise value calculations and duration of franchise value calculations, understand the tradeoff between Franchise value and Duration, and be able to make a supported recommendation for a pricing strategy based on these inputs.
- Generally candidates scored well on the question; most obtained near full credit.
    - It was very common for the candidate to compare duration or franchise value, but not both, in which case, full credit was not awarded

Part a

- The candidate was expected to know formula for Franchise value, and be able to solve for F with the information provided. There were two formulas from the Panning paper that could be used.
- Full credit was awarded if the correct answer was calculated
- Many candidates used the formula F = (P-E-L/(1+y))*d/(1-d), where d=cr/(1+y),and were not able to solve for F given the information provided, as it would require more than direct substitution.
- Calculation errors were common.

Part b
• The candidate was expected to know formula for Duration of Franchise value, and be able to solve for D with the information provided. • Full credit was awarded if the correct answer was calculated
Part c
• The candidate was expected to weigh the tradeoff between franchise value and duration of franchise value, and make a logical recommendation recognizing the goals of both maximizing franchise value and minimizing duration of franchise value. • Full credit was awarded if a recommendation was made which is supported by a correct comparison between two options of franchise value and also duration of franchise value. It must be clear in the response that a higher franchise value is optimal and a lower duration of franchise value is optimal. • Many candidates compared only Franchise value, or only Duration of franchise value. • Many candidates compared target rate of return between the two strategies as support. The graders felt this was missing the important considerations in selecting a pricing strategy, so this was not accepted for credit. If a comparison of franchise value and duration accompanied the comparison of target rate of return, no points were deducted.

QUESTION *11*	
TOTAL POINT VALUE: *2.5*	**LEARNING OBJECTIVE: *B.5***
SAMPLE ANSWERS (BY PART, AS APPLICABLE)	
Part a: *0.5* point	

- **Sample answer #1**

$$D = \frac{(3)2.8M(1.03)^{-3} + (7)(5.2M)(1.03)^{-7} + (20)(7M)(1.03)^{-20}}{2.8M(1.03)^{-3} + 5.2M(1.03)^{-7} + 7M(1.03)^{-20}}$$

$$= \frac{114.80}{10.67} = 10.76$$

assuming discount rate is compounded annually

- **Sample answer #2**

Payment pattern:

T	Payout
0-2	0 %
3	18.67 %
4-6	0 %
7	34.67 %
8-19	0 %
20	46.67 %

$$\text{McCauley Duration} = \frac{3(.1867)\, e^{-.03(3)} + 7(.3467)e^{-.03(7)} + 20(.4667)e^{-.03(20)}}{.1867\, e^{-.03(3)} + .3467e^{-.03(7)} + .4667e^{-.03(20)}}$$

$$= \frac{.5119 + 1.9672 + 5.1226}{.1706 + .2810 + .2561}$$

$$= \frac{7.6017}{.7077} = 10.74 \text{ years}$$

Part b: *1.5* points

- **Sample answer #1**

to immunize, need a portfolio w. same PV (= 10.666M)
and same duration (= 10.763 years)

to get at least 11.6%, will need to use stock (only one w/ return > 11.6%) assuming no capital gains expected, so only the dividend rate is used. Then to get duration equal to 10.763, will need to weight w/ something w/ higher duration. Let's try the 30 yr. bond.

$E(r) = W_{Bond}(0.11) + (1-W_{Bond})(0.143) \geq 0.116$
$\quad = W_{Bond} \leq 0.8182$
$\quad W_{stockD} = 1-0.8182 \geq 0.1818$

Dur. $= 10.763 = W_{Bond}(20) + (1- W_{Bond})(7)$

$W_{Bond} = 0.2895$	PV invested in 20-yr bond = 3.088m	0.2895(10.666m)
$W_{stockD} = 0.7105$	PV invest in stock D = 7.578m	

Using these amounts, the duration and PV will be the same as the liab; so it is immunized against small changes.

- **Sample answer #2**

The only way to get > 11.6% ROR is to invest in stocks. A combination of stock D could get >11.6% and duration of 10.

Stock B & D

$11w + 7(1-w) = 10.763$ $w = .941$
NOPE   annual dividend $= .941(.093) + .059(.143) < 11.6\%$

Stock D & 30 yr bond

$20w + 7(1-w) = 10.76$ $w = .289$ $.289(.11) + .711(.143) = 133\%$

So 28.9% in 30 yr bond + 71.1% in Stock D
$.289(10,666,203) = \$3,082,533$ in 30 yr bond
$\$7,583,620$ in Stock D

- **Sample answer #3**

Use the duration match method it requires
1. PV(liabilities) = PV (Assets)
2. Duration(liabilities) = Duration(Assets)
Only stock D has a return which is higher than 11.6%
So stock D is a must, and we need another securities to adjust the duration.
Stock A is an additional choice with duration of 20 years
It requires:
$W_{stocks} + W_{stockD} = 1$ -> $W_{stockA} = 0.2894$
$20 W_{stocks} + 7_{stockD} = 10.7628$ $W_{stockD} = 0.7106$
The annual dividends of the portfolio is
Dividend $= 10.6662 \times (0.2894 \times 5\% + 0.7106 \times 143\%) = 1.2381$
The stocks are bought at present and sold at end of year 20
IRR (PV = 10.6662, FV=10.6662, PMT=1.2381, n = 20) = 11.61% > 11.6%
Value invested in stock A $= 0.2894 \times 10.6662 = 3.087$ stock D $= 0.7106 \times 10.6662 = 7.579$

- **Sample answer #4**

Based on the given question I decided to use the following 3 securities.
1) 30 year coupon bonds   20 year duration
   (assume WE=a)   11% yield return

2) Stock D: 7 year duration
   (assume weight = b)   14.3% yield

3) Zero coupon bond 20-year duration 20 years
   Yield = 4%   weight = 1-a-b

So I can set the equation set as follow:
$10.763 = 20a + 7b + 20(1-a-b)$

11.6% = 11%a + 14.3%b+ 4%(1-a-b)

By solving these 2 equations I get:
a = 3.99%
b = 71.08%
1-a-b = 24.93%

Therefore PV of liability from (a) = $10,666,203

Therefore at time 0
I should invest =
      10,666,203 x 3.99% = $425,581 in 30-yr coupon bond
      10,666,203 x 71.08% = $7,581,537 in Stock D
      10,666,203 x 24.93% = $2,659,084 in 20-year zero coupon bond

      End up with a asset portfolio duration = 10.763 yrs with investment yield = 11.6%

Part c: *0.5* points

3 yr zero for $\frac{2.8}{1.015^3}$ m = 2.69 m

      7 year zero for $\frac{5.2}{1.025^7}$ m = 4.37 m

      20 year zero for $\frac{7}{1.04^{20}}$ m = 3.19 m

Exact cash flow matching shelters company from reinvestment risk and change in value of bonds since there's no need to rebalance every year or reinvest coupons.

EXAMINER'S REPORT (BY PART, AS APPLICABLE)

Candidates generally performed well on this question.
Candidates were expected to know various strategies for asset-liability matching (ALM) and understand how to apply them to a specific scenario.
Part b was challenging, as it required application and synthesis to make selections from a variety of asset options.

Part a
- Candidates were expected to know how to calculate Macaulay Duration for a series of liability payments.
- For full credit, candidates were expected to demonstrate knowledge of how Macaulay Duration is defined (the correct formula and applicable interest rate) and to calculate the correct result.
- Candidates generally scored well on this part.
- Common errors included applying incorrect interest rates and calculation errors.

Part b
- Candidates were expected to know that a duration matching asset-liability management strategy will immunize against small changes in interest rates and how to apply that knowledge to a selection of investments to design an appropriate portfolio.
- Candidates who received full credit demonstrated the knowledge of duration matching

strategy by selecting appropriate assets for the portfolio that resulted in matching the duration of the liabilities and producing an adequate expected return. They also demonstrated their understanding of asset-liability management by giving sufficient explanation of their approach and correctly calculating the portfolio composition and expected return.

- Multiple portfolios were possible that met the criteria of matching duration and an adequate return. Successful candidates identified that any such portfolio must include Common Stock D along with other asset(s) with longer duration.
- Common errors that candidates made were:
  - Not using a duration-matching strategy (e.g., cash-flow matching)
  - Using assets that did not allow for achieving the target return and/or the appropriate duration
  - Solving for the return of the portfolio instead of the duration
  - Calculation errors

Part c

- Candidates were expected to know that a cash flow matching or dedication strategy would immunize the company against all interest rate changes and to apply that knowledge to design an appropriate portfolio.
- Candidates who received full credit identified the correct strategy as well as the correct investments
- Common errors included:
  - Using an incorrect strategy (e.g., duration matching)
  - Identifying either the correct strategy or investments but not both
- A number of candidates suggested using stocks as the investment strategy. While this reflects the overall recommendation of the Feldblum paper to provide inflation-sensitive assets to match inflation-sensitive P&C liabilities, it is not a strategy that **completely** immunizes against interest rate changes.

QUESTION *12*	
TOTAL POINT VALUE: *2.75*	**LEARNING OBJECTIVE: C8**-Describe the concept of economic capital (or risk capital) in the insurance industry and various methods of allocating the risk capital to business units or lines of business.

SAMPLE ANSWERS

Part a: *1.25* points

- **Sample answer #1**

99% CTE is top 5

99.5% is the 6th largest

Line	99.5%CTE	99.5%VAR		99.5%CTE	99.5%VAR
A	948	650		.0952	.1912
B	7940	2000		.7971	.5882
C	1073	750		.1077	.2206
Total	9961	3400		100%	100%

$$\frac{10,000 + 9,000 + 8,800 + 7900 + 4000}{5} \qquad = \frac{7940}{9961}$$

- **Sample answer #2**

99.5% CTE – only use top five values

Risk	CTE	Percentage
A	948	9.5%
B	7940	79.7%
C	1073	10.8%
	9961	100%

$$\frac{100+985+980+925+850}{5}$$

99.5% VaR – Going to take Rank 5 for each Risk

Risk	Var	%
A	850	14.7%
B	4000	69.1%
C	940	16.2%
	5790	

Part b: *1.0* point

- **Sample answer #1**

- VaR doesn't reflect seventy of ruin, ignore what's happening at the tail of less distribution, not a good allocation base to address comp's concern

- CTE reflects seventy of ruin by looking at the average loss in the fail, thus address comp's concern

- **Sample answer #2**

– CTE this will do pretty good. It looks at the average tail values so it considers the size of the tail in its distribution.
  - VaR will not do well here because the VaR does not consider the size of tail. If we alter the percentile of VaR to coincide with the shortfall risk it may work better.

Part c: *0.5* point(s)

- **Sample answer #1**

Co-CTE measures risk in relation to the aggregate distribution

$$\text{allocation to B} = \frac{7940}{(11,500+9000+9500+8700+7000)15} = \frac{7940}{9100} = .8687$$

using Co-CTE 99.5%

It will allocate more capital to line B since .887 > .7971

- **Sample answer #2**

It more than likely will increase. B makes up a bulk of the total values considering how they compare to the individual so it should get even more.

EXAMINER'S REPORT

Candidates scored well on each part.

Part a

Expected to know basic methods/formulas for VaR and CTE. In general, most candidates knew the basics of the question being asked for part a). Expectations included knowing the difference between Var and CTE calculations while understanding the unique differences between the two methodologies. Common mistakes included calculation errors along with using the "total" column as an additional scenario like A, B, C. 99.5% VaR is largest loss observed at least 0.5% of the time, thus the item in row 5 or row 6 - Technically the 6th item was correct, however we also gave full credit for those that selected the 5th item as the majority of candidates used the 5th item which was probably the result of confusion caused by the events being ranked in ascending order from 1 up rather than descending order from 1,000 down as Goldfarb does in his paper.

Part b

Candidates were expected to know basic difference between benefits of VaR and CTE. Most common deduction was for incomplete explanation and/or providing a general statement with no details on each method. For example, candidate states that VaR specifies value of the shortfall but does not mention that it also does not consider information about the expected value of the shortfall once a shortfall event has occurred. Candidates were expected to be able to highlight the pros/cons of each method in detail by focusing on which method should be used to help support the company's concern.

Part c

Candidates were expected to know co-measure needs to rank order total losses then realize that line B has the largest percent of the total losses so it gets more capital allocation. Candidates did not receive credit if they just restated the numbers from part a) without referring to needing to rank order total losses and then look at the relation of each line to the total losses to figure out whether line B makes up a larger or smaller portion of these losses. To get full credit, candidates needed to clearly articulate the need to look at total losses and show how there was strong correlation with B making up a large percentage of the tail risk for these total losses. As a result, more tail risk would be indicative of

needing to allocate more capital to the line with the largest contribution.

QUESTION *13*	
TOTAL POINT VALUE: *2*	**LEARNING OBJECTIVE:** C.5 – Discuss the development and complexity of financial engineering products such as mortgage-backed securities and other forms of securitization

SAMPLE ANSWERS

Part a: *1* point

- **Sample answer #1**

Purchasing whole mortgages from issuers was done to eliminate regional imbalances in the home mortgage market, effectively nationalizing the market

Issuing pass through securitizations was done to transfer interest rate risk from issuers to those who were more willing to bear it – investors in the financial markets.

- **Sample answer #2**

i) It freed up balance sheets of bank leaders who were previously limited in how much they could lend by their amount of assets. It addressed the issue of limited availability of banks being able to write mortgages.

ii) The issue of being able to resell the mortgages.
Freddie/Fannie bundled and guaranteed non-subprime mortgages that made it easy for investors to buy and diversify.

- **Sample answer #3**

1) Purchasing whole mortgages from insurers helped address increasing the supply of credit.

2) Issuing pass-through securitized mortgages allowed highly rated securities to be provided at attractive yields that investors would buy. Normally these mortgages would be too costly as investors would require too high a premium due to the risk. This addressed the high cost of credit.

- **Sample answer #4**

Whole mortgages – provided liquidity to the housing market where small local savings and loans were unable to meet the demand
Pass through securities – provided further liquidity to the market allowing capital markets to provide funding for home purchases

- **Sample answer #5**

1. Transfer the risk of default from issuer – first innovation
2. Increase the available funds for mortgage – second innovation
Both increase availability of mortgage to the homeowner.

Part b: *1* point

- **Sample answer #1**

Selling run-off business reserves to a reinsurer (or other insurer) in a loss communication. This removes liabilities from an insurers balance sheet allowing them to free up capital to write mare business (which makes insurance more available and cheaper)

Setting up and selling a CAT bond. This securitization product allows the insurer to remove some of their CAT exposure by selling bonds to investors (passing the risk of a covered CAT event to them). This frees up their capital to write more business thus increasing the availability and reducing the price of insurance.

- **Sample answer #2**

i) Similar to a fronting agreement or 100% quota share treaty. Provides capacity to write additional business

ii) Similar to CAT bond. Passes on risk to those with no natural exposure to it; In CAT bond's case, it is CAT exposure. For MBS, it is interest rate risk for diversification.

- **Sample answer #3**

i) WC insurance relieves business of large future points in return for predictable prem upfront. This addresses bankruptcy or business disruption for employers.

ii) CAT bonds-investors willing to take on CAT risk be diversified from rest of portfolio. Addresses high prem & credit risk involved w/ typical reins.

- **Sample answer #4**

Reinsurance: Reinsures purchase bundles of exposures from insurers. This addresses the capacity problem- individual insurers can't afford to hold risk for large losses (high layers, cat risk, etc.)

Cat Bonds: Allows risk to be passed from insurance market to capital market which has much larger risk bearing capacity. The diversification cat bonds offer to investors allows them to accept lower returns than those provided by traditional reinsurance so it also helps with the supply for high layers that often go uninsured otherwise.

- **Sample answer #5**

Purchasing whole mortgages
Reinsurers use sidecars to write treaties. Sidecars are separate buckets of capital managed by existing reinsurers. This allows them to provide CAT capacity where they may otherwise be at their risk limits (ie FL).

Pass-Thru Securities
I think the key here is that FNMA could pass source of the mortgage risk to the broader capital market. This is similar to (re)insurers issuing CAT bonds – allows them to transfer CAT risk out of the insurance space into capital markets.

Both of these risk transfer mechanisms address limited capacity for risk taking.

- **Sample answer #6**

The first point is similar to a residual market mechanism where the govt backs the uninsurable risks in a market place -> providing availability where there would not be otherwise.

Point 2 is somewhat similar to the CAT bond market place because it has filled a demand by investors

(capital market investors vs. other reinsurers) for risky investments that are uncorrelated with the rest of their portfolios

- **Sample answer #7**

Purchase Mortgages = NFIP -> To promote availability of flood coverage. Primary after the coverage + the NFIP "purchases" the risk from the primary.

Pass Through: Citizens + FHCF + Cat Bonds
Citizens originates the policy but cedes coverage to the FHCF. The FHCF has recently issued Cat Bonds to securitize the risk of catastrophic risk. This was done to provide HO coverage to at-risk Florida citizens w/ admitted market coverage is unavailable.

- **Sample answer #8**

i. CAT Bonds – This is analogous to Fannie/Freddie buying whole mortgages. CAT bonds have increased the capital available to insurers. It used to be just reinsurance was available to protect insurers against large costs -> w/ CAT bonds insurers now have the whole capital market from which it can purchase protection. CAT bonds expanded capital available to insurers -> they can write more cat insurance.

ii. CAT bonds enabled insurers to diversify their holdings. CAT bonds are highly uncorrelated with the market, and thus are desirable as a diversification strategy to investors.

EXAMINER'S REPORT

- The candidate was expected to:
  - o Understand the various innovations in the mortgage market, including the reasons they were adopted.
  - o Understand enough about the insurance marketplace to make connections between these mortgage innovations and similar innovations in insurance.
- Candidates generally received partial credit for this question. A small number of candidates received full credit. Candidates scored worse on part (b) than part (a). The biggest problem for candidates seemed to be providing two insurance-related analogies to each of the mortgage market innovations.
- Part (b) was challenging. It was a synthesis question requiring knowledge of the insurance industry.

Part a

- What the candidate was expected to know
  - o -Understand the various innovations in the mortgage market, including the reasons they were adopted.
- How the candidate was expected to respond to obtain full credit
  - o Selling whole mortgages: needed to clearly indicate that this increased mortgage availability or improved liquidity in local markets.
  - o Pass-through securitization: full credit answers fell into two themes. One, the opening of the capital markets to provide funding for the mortgage market, thus increasing mortgage availability (i.e., the issue addressed in the mortgage market). Two, the creation of a new type of high-quality, low-risk investment that provided diversification to investors' existing options (i.e., the issue addressed in the capital markets).

- Common errors made by candidates
  - A fair number of candidates conflated the concepts of pass-through securitization and collateralized mortgage obligation.
    - Pass-through securitization was an innovation prior to the separation of cash flows into tranches, or separation of interest and principal into different CMOs.
    - If the candidate showed understanding of the fact that pass-throughs increased the capital available to support mortgages, but mentioned tranches or seniority of bondholder claims, we only gave partial credit.
  - A number of candidates incorrectly stated that pass-throughs shared the credit risk of mortgages with investors. Fannie Mae and Freddie Mac had an implicit guarantee of bondholder principal by the federal government. Thus they maintained the credit risk.
  - Very few candidates attempted to answer part (a) by listing issues raised by the two mortgage market innovations, rather than those addressed by them. This did not merit partial credit.

Part b

- What the candidate was expected to know
  - Understand enough about the insurance marketplace to make connections between these innovations and similar innovations in insurance.
- How the candidate was expected to respond to obtain full credit
  - FOR EACH innovation in part (a), provide an analogous insurance mechanism, with enough description to indicate they understood the analogy.
  - FOR EACH analogy given, provide the insurance market issue addressed by that mechanism.
- Common errors made by candidates
  - Many candidates gave only one analogous insurance mechanism on part (b).
    - Some of these made clear attempts to relate the single mechanism to each mortgage innovation listed in the question, and received full credit if the analogy was clear, and a market issue was identified.
    - Most only appeared to attempt an analogy to one of the mortgage innovations, and it was not always clear which innovation they were pointing to.
  - Many candidates gave "risk pooling" and "risk sharing" as the analogous insurance mechanisms in part (b).
    - Candidates seemed to be referring to the discussion in BKM (associated with Learning Objective A4), and many gave a good description of the importance of both risk pooling (increases total risk, but decreases average risk) and risk sharing (portions of a risk shared by many investors to diversify the fixed investment amount of each investor).
    - Whole mortgage selling is an example of the insurance concept of risk pooling, and pass-through securitization is an example of risk sharing, and partial credit was given for identifying this.
    - This was not considered an optimal answer, as the terms in and of themselves are somewhat specific to BKM and do not have common usage in the insurance industry.
    - The main reason, however, why partial credit was not given was that the candidates did not clearly state the market issue addressed by these

"mechanisms." Many described the "decrease in average risk" and so forth, which are features of this concept, but the graders felt this did not tie it closely enough to <u>practical</u> examples in the insurance market.

- o Many candidates picked analogous insurance mechanisms, and made good cases for the analogy with the mortgage innovations, but failed to list a market issue that the mechanism resolved. This merited only partial credit.
- o A relatively common answer for part (b) that received no credit was CDOs and CDO-squareds.
- o A less common answer for part (b) that typically received little or no credit was Credit Default Swaps.
- o Many candidates gave Cat Bonds as the analogous insurance mechanism for pass-through securitization (probably the best answer), but some went into too much detail about how cat bonds work (special purpose reinsurer, swap, etc.) without discussing the market issue Cat Bonds addressed. Additional detail that was not necessary to answer the question gained no additional credit.

QUESTION *14*	
TOTAL POINT VALUE: *2*	**LEARNING OBJECTIVE: *C-10***
SAMPLE ANSWERS	

- **Sample answer #1**

rf = .05

r = cost of capital = 15%

undiscounted LR = .7

expenses 25%

premium 43M

C1 – 25M -> released as claims are paid

Expenses paid at the end of yr 1

Required RAROC = $\dfrac{RC_i(1+y)^{-L}}{C_1}$ → investment rate

↓

Target rate of return

A

t	Capital	B Capital
0	25M	25M
1	25M -25M(.6) = 10M	25M -25M(.8) = 5M
2	10M -25M(.3) = 2.5M	5M-25M(.2) = 0
3	2.5M -25M(.1) = 0	0

Discount of using risk free rate 5%   assume annual compounding

t = 0   +43M

t = 1   pay expenses   -.25(43M) = -10.75M ⎤
         pay yr 1 losses   -.6(.7)43M= -18.06M ⎦ -28.81M

t = 2   pay yr 2 losses   -.3(.7)43M = -9.03M

t = 3   pay yr 3 losses   -.1(.7)43M = -3.01M

net income = 43M $-28.81^n(1.05)^{-1}$ $-9.03M(1.05)^{-2}$ $-3.01M(1.05)^{-3}$

= 4.77M

$\dfrac{\text{net income}}{C_1}$ = $\dfrac{4.77M}{25M}$ = 19.09% >15% cost of capital

<21.02% multi yr adj

for y = rf

line A required RAROC = $\dfrac{.15\left(25^M(1.05)^{-1}+ 10M(1.05)^{-2}+ 2.5M(1.05)^{-3}\right)}{25M}$

= .2102

Line B

t = 0   +43M

t = 1   pay expenses -10.75M   34.83M
       pay yr 1 loss   -(.8)(.7)(43M) = -24.08M
       pay yr 2 loss -(.2)(.7)(43M) = -6.02

net income = 43M -34.83M$(1.05)^{-1}$ -6.02M$(1.05)^{-2}$
        = 4.3683M

line B req RAROC $\dfrac{.15\left(25(1.05)^{-1}+ 5(1.05)^{-2}\right)}{25M}$ = .17

      $\dfrac{\text{net income}}{C_1}$ = $\dfrac{4.3683M}{25M}$ = 17.47%   >multi yr RAROC

→ grow line B

- **Sample answer #2**

Line A Capital Commitment	Required Income	PV(time1)
Year 1. 25M	25(M)(.15) = 3.74M	3.75 M
2. 10M	1.5M	1.4286 M
3. 2.8M	.375M	.340136
		5,518,707.5

Line A Income: (43,000,000)(1.05) -.25(43,000,000) −(.7)(6)(43,000,000)1.05^{-1} − (.7)(.1)43M(1.05^{-2})
        = 45,150,000 – 10,750,000 – 18,060,000 – 88,600,000 – 2,230,159
        = 5009841 at time 1

Line B Capital Commitment	Required Income	PV Income (time = 1)
Year 1. 25M	3.75M	3.75M
2. 5M	.75M	.71426M
		4,464,286

Line B Income = 43(M)(1.05) - .25(43M) - .8(.7)25M - .2(.2)(25M/1.05
       = 4,586,667

Since only Line B provides enough income to satisfy the required return on capital requirements, that line should be grown.

- **Sample answer #3**

14.1   <u>A adj. Cost of Capital</u> = .2102
      Formula adj. CoC

CoC x ($\sum\limits_{i=0}$ $\dfrac{\text{capital}_{MG}}{\text{Capital time 0}}$; $(1+rf)^{-1}$)

                  = .15 $\dfrac{\left(25,000,000(1.05)^{-1}+ .4*25,000,000(1.05)^{-2}+ 25,000,000*.1(1.05)^{-3}\right)}{25,000,000}$
                  = .2102

<u>B adj. Cost of Capital = .17</u>

Write B has expected $= .15\dfrac{(25{,}000{,}000(1.05)^{-1} + 25{,}000{,}000*.2(1.05)^{-2})}{25{,}000{,}000}$

RAROC greater than adj. $= .17$
Cost of capital .1747>.17
A's RAROC is less than adj. CoC .1909 < .2102

<u>A expected RAROC</u>
$= \dfrac{4{,}771{,}275}{25{,}000{,}000} = .1909$

<u>B expected RAROC</u>
$= \dfrac{4{,}368{,}254}{25{,}000{,}000} = .1747$

A flows	1	2	3	4
Beg.	Prem	exp.	Loss	disc.
0	43M			
1		.25*43M	.7*43M*.6	$(1.05)^{-1}$
2			.7*43M*.3	$(1.05)^{-2}$
3			.7*43M*.1	$(1.05)^{-3}$

Economic profit A = (1-2-3)*4
= 43M-27,435,095 – 8,190,476 -2,600,151
= 4,771,278

B flows				
Beg.	Prem	exp	loss	dis
0	43M			(1.000)
1		.25*43M	.7*43M*.8	$(1.05)^{-1}$
2			.7*43M*.2	$(1.05)^{-2}$

Economic profit B = 4,368,254

- **Sample answer #4**

Premium = 43
Loss – 43 x 0.70 = 30.1

Loss	Line A	Line B
Year 1	18.06	24.08
Year 2	9.03	6.02
Year 3	3.01	

At T = 1; PV (L$_1$) = 18.06 + $\dfrac{9.03}{1.05}$ + $\dfrac{3.01}{1.05^2}$ = 29.39   E = 0.25x 43 = 10.75

PV (L$_2$) = 24.08 + $\dfrac{6.02}{1.05}$ = 29.81

Capital	Line A	Line B
Year 1	25	25
Year 2	25 x (1-0.6) = 10	25(1-0.8) = 5

Year 3   25 x (109.9) – 2.5

PV Capital $_1$ = 25 + $\dfrac{10}{1.05}$ + $\dfrac{25}{1.05}$ = 36.79, PV Capital $_2$ = 25 + $\dfrac{5}{1.05}$ = 29.76

RAROC   A = $\dfrac{P(1+1) - E - PV(L)}{Capital}$ = $\dfrac{43(1.05) - 10.75 - 29.39}{36.79}$ = 13.63%

RAROC   B = $\dfrac{43(1.05) - 10.75 - 29.81}{29.76}$ = 15.42%

Line B should grow because has RAROC greater than Cost of Capital   15.42% > 15%

- **Sample answer #5**

A = PV $_{t=1}$ (losses) = .6(.7*43M) + .3(.7*43M)(1.05^{-1}) + .1(.7*43M)(1.05^{-2})
          = $29.39M
Net Income @ time 1 = $43M(1.05) – 43M(.25) – 29.39M
          = $5.01 M
    Cost of Capital = .15(25M) + .15(25M*.4)(1.05^{-1}) + .15(25M*.1)(1.05^{-2})

                          25M-25M(.6), since 60% capital released after yr 1
          = $5.52 M
EVA = Net Income – Cost of Capital = -$0.51M

B = PV $_{t=1}$ (losses) = .8(.7*43M)+/2(.7*43M)(1.05^{-1})
          = $29.81M
  Net Income $_{t=1}$ = $43M(1.05) – 43M(.25) -29.81M
          = $4.59M
  Cost of capital = .15(25M) + .15(25M*.2)(1.05^{-1})
          = $4.46M
      EVA = 4.59M -4.46M = $0.13M

Because A has a negative EVA, it is eroding firm value. Because B has positive EVA, it is increasing firm value. Company should hence grow Line B.

EXAMINER'S REPORT

The candidate was expected to know how to compute the Cost of Capital and the Economic Profit for each line and make the correct decision based on these results. Full credit was also given when computing and comparing the Target and Expected RAROC.

Candidates had difficulty on the question as a whole. The majority of candidates were able to compute either the Cost of Capital or the Economic profit but often did not do both. This led to a flawed decision process used to choose the correct line to grow.

Common errors included:
- Computing the target RAROC for each line without computing the expected RAROC and comparing the targets to each other
- Computing the expected RAROC for each line and without the target RAROC and comparing to 15% or to each other
- Inconsistent discounting periods for losses, economic profit and/or capital: for example, discounting the Cost of Capital to time 0 but discounting the Economic Profit to time 1
- Handling expenses incorrectly
- Computing the IRR for each line
- Using the wrong discount rate

QUESTION 15	
TOTAL POINT VALUE: 2	**LEARNING OBJECTIVE: LO C1 KS a-e & h)** **"Estimate the credit risk due to default and default correlation associated with fixed income securities."**
SAMPLE ANSWERS	

- **Sample answer #1**

Bond initially rated Aa has .01 prob of being Baa or worse by yr end
Bond initially rated A has .05 probability of being rated Bad or worse by year end

Initial rating	yr end Aaa	Aa	A	Baa worse
Aa	0 to .05	.05 to .95	.95 + .99	.99 to 1
		.05 +.9	.95 +.04	
A	0	0 + .05	.05 + .95	.95 to 1
			.05+.9	

.99 corresponds to Z = 2.326
.95 corresponds to Z = 1.645

If Z_1 is Aa bond and Z_2 is A bond
 Then we are looking for $Z_1 \geq 2.326$
$$Z_2 \geq 1.645$$
$$= 1366 + 570 = 1936$$

 From the other direction Z_1 is A and Z_2 is Aa
$$Z_1 \geq 1.645$$
$$Z_2 \geq 2.326$$
$$= 1275 + 570 = 1845$$

 So range is $\dfrac{1845 - 1936}{1M}$ =

.1845% to .1936%

- **Sample answer #2**

Aa -> Baa or worse = 0.01
A -> Baa or worse = 0.05

$\Phi^{-1}(0.01) = 2.326$
$\Phi^{-1}(0.05) = 1.645$

$z_1 < -2.326$	$z_2 < -1.645$:	556+ 1.261 = 1,817 min
$z_1 < -1.665$	$z_2 < -2.326$:	556 + 1.333 = 1,889
$z_1 > 2.326$	$z_2 > 1.665$:	1,366 + 570 = 1,936 max
$z_1 > 1.665$	$z_2 > 2.326$:	1,275 + 570 = 1,845

Therefore, range = (0.1817%, 0.1936%)

- **Sample answer #3**

Φ^{-1} (P(Aa bond will be Baa or more)) = Φ^{-1} (0.01) = 2.326
Φ^{-1} (P(A bond will be Baa or more)) = Φ^{-1} (0.05) = 1.645

If z_1 corresponds to Aa & z_2 to A
- → P(both bonds will be Baa or more) = $\frac{556+1261}{1,000,000}$ = 0.001817

If z_2 corresponds to Aa & z_1 to A
- → P(both bonds will be Baa or more) = $\frac{556+1333}{1,000,000}$ = 0.007889

Range –> 0.1817% to 0.1889%

EXAMINER'S REPORT

- What the candidate was expected to know
  - The candidate should be able to correctly interpret the transition matrix to understand that there is a 0.05 probability that the A-rated bond will be Baa or worse at year end and there is a 0.01 probability that the Aa-rated bond will be Baa or worse at year end.
  - The candidate should be able to correctly map the two probabilities to the appropriate Z-values via the normal distribution.
  - The candidate should recognize that the simulated bivariate normals can be interpreted in different ways to produce distinct estimates of the downgrade probability.
  - The candidate should understand how to use the simulated values to estimate the range of probabilities
- Responses commonly reflected correct calculations for one point of the range but then the candidate was not sure how to determine the other end of the range. Often this seemed due to the candidate not realizing Z1 and Z2 could be either the Aa bond or the A bond.
- The most common mistake was when developing the range. Often, candidates only provided one of the possible simulated estimates. Additionally, those that determined one of the possible simulations had various approaches to putting a range around one value, even though this one value represented an endpoint of the correct range.
- The most challenging part of the question was interpreting the simulation to form the range.

QUESTION 16	
TOTAL POINT VALUE: 3.5	**LEARNING OBJECTIVE: C-3, C-5**
SAMPLE ANSWERS	

Part a: 0.5 point

- **Sample answer #1**

The CAT Bond would be preferable b/c CAT Bond risk is uncorrelated with market risk, while the risk for CDOs is highly sensitive to market risk.

- **Sample answer #2**

The market expects the same return on all 3 securities. According to CAPM, expect return is a   function of its covariance w/ the market
  - The CAT bond is largely uncorrelated with the market
  - The junior tranch is not dominated by systematic risk
  - But the sr tranch of the CDO^2 is very much exposed to systematic risk

CAPM would suggest a greater expected return by its correlation w/ systematic risk
 -The CAT bond has the least risk (& same expected return) so it's preferred according to CAPM.

Part b: 1.5 points

- **Sample answer #1**

CAT bond is individual not correlate with other bond if prob default
Increase prob of default of it increase equal to prob
Junior tranche absorb first losses if prob of default increase
It is affected very much -> most
Senior tranche (CDO^2) -> last absorb losses after junior and mezzanine
It is affect at least

 ->most preferable   senior tranche CDO^2 > CAT bonds > junior

Part c: 1.5 points

- **Sample answer #1**

Correlation going ↑ could ↑ price of junior tranche since prob of 2 defaults would approach prob of 1 default. Senior tranche of CDO^2 would go down in price because senior tranche was previously providing diversification benefits (in case one junior tranche defaults but not other). Higher corr. decreases divers. benefit and decreases price. CAT bond price should be unaffected since it only has a single underlying risk.

- **Sample answer #2**

Cat bond – no effect unless only 3 securities in world and mortgage changes effect demand/optimum portfolios.

Junior tranche – price goes up because risk shifts to higher tranches.

 Senior –price goes down as risk increases from correlated defaults.

EXAMINER'S REPORT

Part a

Full credit was received by anyone who responded that the catastrophe bond was preferable because it has the least systematic risk.

Candidates lost points for replying that all securities were equally preferable according to CAPM because they all had the same expected return. This was an unacceptable answer, as it ignores the varying levels of systematic risk of the three types of securities, and the fact that the return expected by the market is not the same as the theoretical return suggested by the CAPM formula.

Another common error was to choose the security with the <u>highest</u> level of systematic risk. While CAPM does reward high systematic risk with a higher expected return, because the securities all have the

same expected return, the security with the highest systematic risk would actually be the worst investment (it would have the lowest alpha).

Some candidates lost points for explaining that the security with the lowest systematic risk was preferable, but then choosing the CDO or CDO^2 instead of the cat bond.

Part b

Candidates struggled with this part. The key point is that as the probability of default increases, the junior tranche of the CDO would be the most impacted, as that tranche is the first to suffer losses, while the senior tranche of the CDO^2 would be insulated from the increased level of defaults. The cat bond, meanwhile, would suffer proportionally to the increase in default.

Many candidates failed to make this connection, or failed to explain their rankings. Candidates were expected to not simply list the preferred order of securities- a valid explanation is required. Any ranking with a valid explanation was accepted.

A common error was to state that the CDO^2 is least preferable because "errors in default estimates are magnified by the CDO^2 structure". While this may be true in a general sense, in this particular scenario with the junior tranche suffering the first losses and shielding the CDO^2, it is not relevant.

Many candidates tried to demonstrate which security was preferable by showing calculations of default probabilities for each security, or explaining which security was least likely to default in general. This ignored the aspect of the <u>increase</u> in default probability, its relation to the market price of the securities, as well as the tranche structure of the CDO and CDO^2 as given in the problem.

Some candidates responded that cat bonds would be the most preferable as they are fully collateralized. All 3 securities are collateralized, and credit risk should not be a consideration in this problem. Another common theme was to bring up bond ratings. While the CDO^2 would most likely suffer a rating downgrade in light of increased default rates, the CDO and cat bond would also suffer, and there is not enough information given in the problem to state definitively which instruments would become more or less preferable as a result of ratings changes.

Part c

This part was similar to Part b, but candidates handled it a little better. Since the part only asked for impact on market prices, as opposed to a ranking of securities, it involved a little less synthesis. Most candidates handled the cat bond well, as it was unaffected by the correlation of mortgages. The CDO and CDO^2 were a little more difficult, as it is fairly unintuitive that the junior tranche of a CDO would actually increase in market price due to a rise in correlation. It seemed easier for most candidates to see that the senior tranche of a CDO^2 would decrease in market price.

Many of the errors made in Part b were again seen here. While many candidates seemed to know the general impact on market price of the CDO and the CDO^2, many candidates failed to explain why (again, simply stating that "the price goes down because it becomes riskier" is not an acceptable answer).

Another common error here was to recognize that risk shifts from the junior to senior tranche, but then conclude that the senior tranche then becomes more expensive. Candidates seemed to be confused about how risk impacts the price of securities on the open market (or perhaps under time pressure they simply mixed up price and expected returns).

Another common error was to assume that the overall probability of default increased as a result of the increase in correlation. However, when the correlation increases, it does not affect the default probability of a single mortgage, it simply makes it more likely that mortgages will default at the same time.

QUESTION 17	
TOTAL POINT VALUE: 1.75	**LEARNING OBJECTIVE: C.7**
SAMPLE ANSWERS	

Part a: 1 point

Solution 1

> With stock options owners have asymmetric gain/loss profiles. They gain much from improved performance but lose little on the downside. Thus they are incented to use less risk mgmt (hedging) to increase volatility in hopes of greater upside potential in their compensation.
>
> With direct ownership, owners gain and lose along with the company and thus desire stability over volatility. Thus we might expect increased hedging to protect against downside risk and loss of value to the managers.

Solution 2

> Stock options generally lead to more risky/aggressive decision making as they will only pay off with large increases in firm value. When the risky decision turns out poorly, the option is worth the same (0) as if management took no risks and steadily (but slowly) grew the business. When ownership of shares is the incentive, management has the incentive to avoid risks as a large portion of their assets are tied to this non-diversifiable risk.

Part b: 0.75 point

Solution 1

> If the company is in the midst of severe financial distress –> most of the remaining value of the company will go to the debt holders, not the shareholders. At this point, the shareholders will encourage risk, as the upside means bringing the company out of distress and increasing the equity value, while the downside is limited since most of the money will go to debt holders anyway. Here the stock options are preferable, since this encourages management to place bets in hopes of the option paying off.

Solution 2

> Stock options better aligns. In this situation there is little downside in taking risks since the company is already distressed. If the risk pays off then it may help save the company from bankruptcy.

EXAMINER'S REPORT

- The candidate was expected to understand the compensation structures for stock options and direct ownership of shares. The candidate was also expected to understand the impact of these compensation structures on the level of Risk Management employed by the company as well as the level of risk management desired by shareholders of a financially distressed company, as outlined by Stulz.
- Candidates scored generally well on this question as a whole, though they had more difficulty with part a) as described below and part b) required some synthesis from concepts presented in the same paper.

Part a

- The candidate was expected to understand how the compensation structures worked and their effect on risk management utilization.
- To obtain full credit, the candidate was expected to explain fully the way that each compensation structure specifically impacted risk management. It was important for the candidate to highlight how each compensation structure caused this effect in a way that was distinctly different from other structures.
- Often candidates neglected to fully discuss the impact on risk management in ways that differentiated that particular compensation structure. For instance, a number of candidates focused solely on direct ownership as giving management a stake in the company's results and/or management being rewarded for positive results by direct ownership; however the same can be said of management under the stock option pay structure or simply salary. All employees of a company have some stake in the company's success.
- A number of candidates merely described the effects of the compensation structures without adequately discussing how the structure led to that effect.

	• Many candidates did not mention the difference in the compensation structures' handling of the downside possibilities of risk; this was a key point in demonstrating a full understanding of the learning objective.
Part b	
	• The candidate was expected to understand the level of risk management that one would expect shareholders to desire in the face of severe financial distress. The candidate was also expected to understand how the compensation structure for management could help guide management toward the shareholders' desired direction. • To obtain full credit, the candidate was expected to correctly choose the stock options compensation structure and to explain fully the reasons shareholders should desire less risk management in the given scenario in such a way that demonstrated an understanding of the uniqueness of the financially distressed situation. • Often candidates did not articulate shareholders' general disregard for the downside possibilities of risk in this situation due to the already dire situation faced by the company. These candidates would focus on the upside possibilities of taking risk, but to demonstrate a full understanding of the concept, the candidate needed to recognize the uniqueness of the downside risk in the scenario.

TOTAL POINT VALUE: *3*	LEARNING OBJECTIVE: *D1*

SAMPLE ANSWERS

Part a: *2.5* points

*Assuming no taxes

	0	**1**	**2**
Premium	125M	0	0
Expenses	35M	0	0
Loss Paid	0	70.3125M	23.4375M
Loss Reserve	93.75M	23.4375M	0
Surplus	62.5M	15.625M	0
UW Income	-3.75M	0	0
Inv. Income	0M	6.25M	1.5625M
Net Income	-3.75M	6.25M	1.5625M
Δ Surplus	-62.5M	46.875M	15.625M
Cash Flow	-66.25M	53.125M	17.1875M

(Loss reserve & surplus) x 0.04

$$0 = -66.25M + \frac{53.125M}{1+IRR} + \frac{17.1875M}{1+IRR}$$

IRR = 4.92%

Certain candidates used a slightly modified approach of the above and calculated the change in surplus, reserves, losses and investment income in order to get to the equity flows. This was also an acceptable response.

Part b: *0.5* point

Solution 1

Equity providers are expecting a target return which is above what they could normally get by just investing. This comes in the form of excess investment income on reserves and underwriting profit. If rates are inadequate they may not get their return so the will pull money out of the company.

Solution 2

Rates are not adequate, as the IRR (4.92%) is less than the cost of capital of 7%. Equity providers can pull out their capital and invest alternatively at 7% unless the insurer raises their rates such that IRR >= 7%.

EXAMINER'S REPORT

Part a

The candidate was expected to at least be able to calculate losses, reserves, surplus, expenses and place them at the correct time as well as setup an IRR equation and solve it. The parts that candidates missed more frequently were the investment income (Required Assets * Investment

Yield) and the equity flows, but these were still very well answered in general.

A large proportion of candidates got full credit or near full credit – common mistakes being simple calculation errors, or lack of detail in the response (not showing important steps of the calculation, such as investment income).

Part b

Candidates performed very well on this sub-part which was answered with full credit by a very large majority of candidates. Candidates were expected to know that cost of capital is a benchmark for rate adequacy from an equity provider perspective.

Note: Grading was adapted to the IRR calculated in part A.

QUESTION 19	
TOTAL POINT VALUE: 2.25	**LEARNING OBJECTIVE:** D5
SAMPLE ANSWERS	

Part a: 1.25 points

Solution 1

$U = U^o - PLR [PVxo - PVx]$ assume annual compounding

$U = 2.5\%$

$U^o = 5\%$

disct at new money rate

yr	ref line	rev line	PV	(a)x(c)	(b)x(c)
1	.5	.1	$(1.04)^{-1}$	.4808	.0962
2	.3	.4	$(1.04)^{-2}$	.2774	.3698
3	.2	.3	$(1.04)^{-3}$	.1778	.2667
4	0	.2	$(1.04)^{-4}$	0	.1710
				.9359	.9036

$.025 = .05 - PLR [.9359 - .9036]$

$PLR = .7740$

Part b 1: 0.50 point

Solution 1

We used the new money rate to discount the payment patterns. These rates will apply on a prospective basis so the new money yield is more relevant than what the company has earned on investments in the past.

Solution 2

I use the new money rate of return as the discount rate. Though there exists opportunity cost for policyholders, they have no right to claim for the investment return excess the risk free rate. Sure they don't pay for loss in the investment, which is burdened by equity holders. Thus I don't use the investment rate of income, but the new money rate of return as the discount rate.

Solution 3

We used the historical investment rate because it is stable throughout time and so not much volatility and it's also variable through our investor relations and annual filings.

Part b 2: 0.50 point

Solution 1

I don't think the use of target return on surplus that was used to generate the filed rates is a good thing. Because if use the target return on surplus to generate filed rates, it will make the insurer with huge capital get lower rate. However, the insurer with huge capital provides more protection for the insurance policy written. It's not equality.

It's better to use the target return on sales (premiums) to generate the filed rates.

Solution 2

$$\frac{T}{S} = \frac{I}{A}\left(1 + \frac{R}{S}\right) + \frac{U}{P} \times \frac{P}{S}$$

The target return on surplus was not used to determine our underwriting profit loads and therefore is not valid in determining if our rates are adequate. This method to determine profit load is based on previously accepted U/W profit load but we adjust for the time income can be

earned on the P/H surplus.

EXAMINER'S REPORT

The present value offset method is a relatively well tested approach and candidates generally scored well on this question, many obtaining full credit. While the present value offset method may not reflect the most common way ratemaking actuaries compute a profit and contingencies provision, it requires the well-prepared candidate to understand the core concepts which make it a requirement for rates that are not inadequate, not excessive, and not unfairly discriminatory. For this particular question, the candidate is expected to quantify the impact of changes to insurance risk duration on a preset profit load by comparing the cash flows of two lines of business: one with a longer payout pattern and one with a shorter payout pattern. The question then asks the candidate to formulate a cogent argument regarding the discount rate used as well as the target return on surplus.

Part a

This part of the question asked candidates to calculate the profit provision required by the new line of business. Candidates generally scored well, with most identifying the present value offset method as the necessary approach. The majority of mistakes fell in to two distinct buckets:

1. Using the incorrect discount rate, or
2. Making a single computation error.

Candidates that did not show all the calculation steps necessary to derive their final answer were sometimes given less credit as their methodology and approach could not always be verified.

Part b

Part b has two subsections. For part b1, which asked the candidate to identify and justify the discount rate used, candidates scored well. Part b2, which asked the candidate to disclose what return on equity/surplus assumption was used in their calculations, is where candidates struggled. Regarding the Present Value Offset Method, a key advantage to the approach is that there is no explicit ROE/ROS requirement. Many candidates replied as such. However, by synthesizing other areas of the syllabus (particularly McClenahan), candidates were given credit for other responses (as illustrated in the model solutions).

QUESTION 20	
TOTAL POINT VALUE: 2.5	**LEARNING OBJECTIVE:** A.4
SAMPLE ANSWERS	

Part a: 1.5 points

Solution 1

- Corporate Target underwriting profit
  - o UW profit used for corporate decisions of what rates to file to hit target returns
- Breakeven UW Profit provision
  - o This is U/W profit required to earn risk free return for investors (no compensation for risk).
- Charged UW Profit provision
  - o This is charged UW profit after experience rating and schedule modifications are made (Robbin).

Solution 2

Break even profit – calculated so the insurer will just "break even" – set profit = risk free rate of return

Manual profit – calculated using manual rates

Accrual profit – accrual UW profit almost always differs from the priced profit provision due to catastrophes and other unexpected losses or market events during the policy period.

Solution 3

Statutory – Earned premium less incurred expenses and undiscounted loss

GAAP – Earned premium less deferred expense and undiscounted loss

IASB fair value - Earned premium less deferred expense and discounted loss

Solution 4

1. Underwriting profit due to risk selection. UW profit due to good UW control, risk control and selection.

2. Underwriting profit due to expense saving. UW profit due to improved company efficiency or expense spending.

3. Underwriting profit due to improving loss experience. UW profit due to improved loss experience coming from good loss control and risk mitigation plans.

Part b: 1.0 point

Solution 1

The split between policyholder supplied funds (premiums and reserves) and stockholder supplied funds (surplus) is somewhat arbitrary because it depends on the insurer's reserving practices today and in the past, whether a risk margin is added to reserves, whether expenses are priced for, etc. So, the split between "PHSF" and "surplus" isn't really a definite, actual split, it's just an arbitrary construct. It's fairer to just include inv. Income in all funds an insurer has.

Solution 2

A lower target profit to include II on surplus b/c:

1- When the actual UW credit > Target, the insurer and the investor benefit w/higher returns but the insured does not. Thus including II on surplus will help offset this.

2- It will help maintain that capitol is being used efficiently if insurers are overcapitalized they will have more II on surplus and thus lower on profit, this will encourage them not to be

overcapitalized. The extra capitol can be deployed in other assets to ensure economic efficiency.

3- It will obviously benefit consumers by lowering the price that they are charged for coverage.

Solution 3

Not sure if the question meant "shouldn't." If so, here's the answer.

- o Usually, it would not include the investment income on SSF because this income doesn't belong to policyholders. Including it means that policyholders are getting extra protection for less cost. The surplus does not belong to them.

- o An argument to include allowing profit provision: It should be included as heavily capitalized insurers might be earning much more than their competitors since they can still charge same rate than the others by showing that they have low U/W income (excluding the investment income for S/H) and still have a much better return on surplus. Therefore, if rate regulation is based on return on surplus, it should be included.

Solution 4

1. Investment income earned on stockholders supplied funds (SHSF) are real earned profit by the insurer.
2. The distinction between policyholders supplied fund (PHSF) and SHSF are somewhat artificial as they are not visible on financial/annual statements to the regulators.
3. Any claim is backed up by the full amt. of the insurer's surplus, if there's any deficiency in prem. charged and investment income earned on PHSF the SHSF will have to cover the shortage.

EXAMINER'S REPORT

This question tested the candidates' knowledge regarding UW Profit and the UW Profit Provision. This question stemmed from the Robbin paper, although accurate answers that were not from that paper were accepted and given credit.

Part a

The most common error was describing three methods of calculating the UW Profit Provision, rather than three types of UW Profit. Another common error was describing other types of profit, i.e., not UW Profit.

Part b

This part was designed to have the candidate formulate an argument for something that is generally not done. If the candidate argued the opposite (i.e., that Investment Income on Shareholder Supplied Funds should NOT be included in the UW Profit Provision calculation), they still received credit as long as they specifically stated what they were arguing. There were a number of reasons that could be provided (for either argument). The candidate only needed to have one reason, but the expectation was that they would flesh that reason out, showing pros and cons about that reason, rather than just writing a sentence stating the reason. Partial credit was awarded, however for stating the reason.

Common errors were candidates referring to Policyholder supplied funds (rather than Stockholder supplied), stating an invalid reason in their argument, or a reason that was for the opposite argument (without specifically stating they were choosing the opposite argument).

QUESTION 21	
TOTAL POINT VALUE: 1	**LEARNING OBJECTIVE: D.6.**
SAMPLE ANSWERS	

Part a: 0.5 point

<u>All Surplus – Solution 1</u>

     The insurer's entire surplus supports all of the insurer's business, regardless of state or line, so this allocation is arbitrary.

<u>All Surplus – Solution 2</u>

     All its surplus stands behind all its business – company can't go insolvent by line or by state.

<u>Correlation – Solution 1</u>

     The risk of each line interacts with other lines. To allocate by P/S ratio may overstate the surplus in total.

<u>Correlation – Solution 2</u>

     The line of business is correlated with sources of risk, so the total surplus may be less than the sum of surplus of each line.

<u>Premium – Solution 1</u>

     Many risks (pricing, reserving, credit, CAT risk) are not proportional to prem. Thus multi-line insurers will be exposed to many different risk and using P/S to allocate surplus is not appropriate.

<u>Premium – Solution 2</u>

     The risks that the insurer is exposed to are not necessarily proportional to premium. It might be inappropriate to allocate based on premium to surplus ratio.

Part b: 0.5 point

<u>Wide range of risks – Solution 1</u>

     Different insurers are exposed to different types of risks. It's not proper to use industry ratio to allocate.

<u>Wide range of risks – Solution 2</u>

     Individual insurers can have very different risk profiles – what may be appropriate for one insurer may not be for another – e.g. a monoline workers compensation carrier in CA vs a multi-line, national insurer that writes in CA

<u>Unique risks – Solution 1</u>

     Insurance industry as a whole: each company w/in the industry has their own unique risks and one prem to surplus ratio is not appropriate for all.

<u>Unique risks – Solution 2</u>

     There is no one P:S appropriate for the industry. Each insurer has a different set of risk and therefore has a different P:S ratio for its risks.

<u>Unique risks – Solution 3</u>

     Each insurer writes different mix of business and is exposed to different risks. So surplus allocated should reflect the level of risk inherent in each insurer, and premium alone does not reflect this.

<u>Premium – Solution 1</u>

     There are risks in insurance industry that are not associated with premium, such as asset risk.

<u>Premium – Solution 2</u>

> The insurance industry exposes many risk sources, e.g., credit risk, catastrophic risk, strategy risk, etc, some of the risk is independent of premium.

<u>Premium – Solution 3</u>

> The risks of the industry may not be proportional to premium. i.e. reserve risk

EXAMINER'S REPORT

- The candidates were expected to know the pitfalls of allocating surplus to state and line using Premium to Surplus ratios for both an individual multi-line insurer and the insurance industry as a whole.
- Candidates seemed to have more trouble with part (b)

Part a

- The candidate was expected to know why allocation of Surplus to state and line using P/S ratio may be problematic for a multi-line national insurer.
- Common Errors
  - o Assuming there is single P/S ratio to be applied to all state/lines.

Part b

- The candidate was expected to know why allocation of Surplus to state and line using P/S ratio may be problematic for the insurance industry as a whole.
- Common Errors
  - o Assuming there is a single P/S ratio to be applied to all state/lines.
  - o Assuming total industry surplus is available to any state/line
  - o No specific mention of differences in risk among various insurers.
  - o Responses focused on rate of return and regulation related issues.

QUESTION 22	
TOTAL POINT VALUE: 4.25	**LEARNING OBJECTIVE: LO D9**
SAMPLE ANSWERS	

Part a: 2.25 points

<u>Solution 1</u>

ABC + Part A

r_{ABC} = λ_{mv} x (σ_{ABC}^2 + CovShare$_{ABC}$) = .005 [2538.69 + 26.009]
= 12.82

r_a = λ_{mv} x (σ_A^2 + CovShare$_A$) = .005 [15.92 + .9277]
= .084

$r_{A + ABC}$ = λ_{mv} x (σ_{A+ABC}^2) = .005 x 2581.54 = 12.908

<u>Fill in Cov Share table:</u>

Event 1: $W_{ABC} = \frac{E[L_{ABC}^1]}{E[L_{ABC}^1] + E[L_A^1]} = \frac{350}{352} = .9943$

CovShare$^1_{ABC}$ (ABC,A) = W$^1_{ABC}$ x 2 x E[L$_{ABC}$] x E[L$_A$] x p$_i$ x (1-p$_i$)
= .9943 x 2 x 350 x 2 x .01 x (1-.01)
= 13.781

Event 3: W_A^3 = 40/(40+1) = .9756
CovShare^3_A = .9756 x 2 x 1 x 40 x .01 x (1-.01)
= .77268

Total CovShares:
CovShare$_{ABC}$ = 13.781 + 3.91 + …. + 6.562 = 26.009
CovShare$_A$ = .079 + .01 + .7727 + .009 + .057 = .92787

ABC + Part B

r_{ABC} = .005 x (2538.69 + 1372.788) = 19.557

r_B = .005 x (383.13 + 541.762) = 4.624

$r_{ABC + B}$ = .005 x 4836.37 = 24.182

Table:

Scenario	ABC	Added Part	Total
ABC + A	12.82	.084	12.908
ABC + B	19.557	4.624	24.182

<u>Solution 2</u>
CoVar share method:

Risk Load = λ*[var(addition) + covshare (existing, addition)]

ABC covshare event 1 = $\frac{350}{350+2}$ x 2 x (0.01)(1-0.01)(350)(2) = 13.781

Portfolio A covshare event 3 = $\frac{40}{40+1}$ x 2 (0.01)(1-0.01)(1)(40) = 0.773

Scen: ABC + A
Total ABC covshare w/ Part A = 13.781 + 3.91 + 0.019 + 1.737 + 6.562 = 26.009
Total part A covshare = 0.079 + 0.010 + 0.773 + 0.009 + 0.057 = 0.928

Scen ABC + B
Total ABC covshare w/ Part B = 1,372.788 (given)

Covshare ABC & B =   4836.37   -   2538.69   -   383.13   =   1914.55
                    Var (ABC + B)   var (ABC)   var(B)   2*cov(ABC,B)

Total portfolio B covshare = 1914.55 -1372.788 = 541.762

Scenario ABC + A: renewal R_{ABC} = λ*[var(ABC) + covshare$_{ABC}$ (ABC, A)]
                        = 0.005 [2538.69 + 26.00(] =   12.823
                    R_A = λ*[var(A) + covshare$_A$ (ABC, A)]
                        = 0.005 [15.92 + 0.928] =   0.084
                    Total= R_{ABC} + R_A =   12.907

When using cov share, risk loads are renewal additive.
Scenario ABC + B: R_{ABC} = 0.005 [2538.69 + 1372.788] =   19.557
            R_B = 0.005 [383.13 + 541.762] =   4.624
            Total = 19.557 + 4.624 =   24.181

Solution 3
under renewal, assume each company is the last in/written
        Risk Load$_x$ = λ(Var(x) + 2*w$_x$*cov(x,y))
ABC & B:
        Risk Load$_{ABC}$ = 0.005(var(ABC) + covshare(ABC, B))
                = 0.005 (2538.69 + 1372.788)
                = 19.557
        Risk Load$_B$ = 0.005 (383.13 + 541.762)
                = 4.624
        Risk Load$_{ABC + B}$ = 19.557 + 4.624 = 24.182
ABC & A:

Event	Cov Share(ABC)	Cov Share(A)
1	*13.781	0.079
2	3.910	0.010
3	0.019	^ 0.772
4	1.737	0.009
5	6.562	0.057
	26.009	0.927

2*Cov (ABC, A) = Var(ABC+A) – var(ABC) – var(A)

$\qquad$ = 2581.54-2538.69-15.92

$\qquad$ = 26.93

$\qquad$ * = $(\frac{350}{352})$ $((.01)(.99)(352)^2 - (.01)(.99)(350)^2 - (.01)(.99)(2)^2)$

$\qquad$ ^ = $(40/41)((.01)(.99)(41)^2 - (.01)(.99)(1)^2 - (.01)(.99)(40)^2)$

$\qquad$ Risk Load$_{ABC}$ = 0.005 (2538.69 + 26.009) = 12.823

$\qquad$ Risk Load$_A$ = 0.005 (15.92 + 0.927) = 0.084

$\qquad$ Risk Load$_{ABC + A}$ = 12.823 + 0.084 = 12.908

Scenario	ABC	Added	Total
ABC + A	12.823	0.084	12.908
ABC + B	19.557	4.624	24.182

Solution 4

λ = 0.005

Renewal risk load for risk$_i$ = λ*(var$_i$ + cs$_i$) = .005 (var$_i$ + cs$_i$)

$$CS_{L1} = \Sigma L_i * n_i * p_i * (1-p_i) * 2 * \frac{Li}{Li+ni}$$

Solve for <u>unknowns</u>

ABC event 1: 350 x 2 x 2 x .01 x .99 x $(\frac{350}{352})$ = 13.78

Portfolio A event 3 = 1 x 40 x 2 x .01 x .99 x $(\frac{40}{41})$ = 0.7727

ABC CS with A = ΣCS_i = 13.78 + $\Sigma^\#$ in table = 26.008

A CS with ABC = 0.7727 + $\Sigma^\#$ in table = 0.9277

a) <u>Scenario</u>   <u>ABC</u>   <u>Added</u>   <u>Total</u>

ABC + A   1   2   3 ⎤

ABC + B   4   5   6 ⎦   see below

1: .005 (2,538.69 + 26.008) = 12.82   4: .005 (2,538.69 + 1,372.788) = 19.58

2: .005 (15.92 + .9277) = 0.0842   5: .005 (383.13 + 541.762 = 4.6245

3: (1)+ (2) = 12.91   6: (4) + (5) = 24.18

Solution 5

covariance share: ABC & A

Event	L$_{ABC}$	L$_A$	%ABC	CovShare$_{ABC}$	CovShare$_A$	Cov Total	
1	350	2	99.432%	13.825	0.079	13.904 =	$\frac{0.079}{1-99.432\%}$
3	1	40	2.439%	0.019	0.76	0.779	

Scenario ABC + Portfolio A:

$R_{ABC} = \lambda*(\sigma_{ABC}^2 + \text{cov share}_{ABC})$

   $= 0.005 \times (2538.69 + 13.825 + 3.91 + 0.019 + 1.737 + 6.562)$

   $= 12.824$

$R_A = \lambda*(\sigma_A^2 + \text{cov share}_A)$

   $= 0.005 \times (15.92 + 0.079 + 0.01 + 0.76 + 0.009 + 0.057)$

   $= 0.084$

$R_{total} = R_{ABC} + R_A = 12.824 + 0.084 = 12.908$

Scenario ABC + Portfolio B:

$R_{ABC} = \lambda*(\sigma_{ABC}^2 + \text{cov share}_{ABC})$

   $= 0.005 \times (2538.69 + 1372.788)$

   $= 19.557$

$R_B = \lambda*(\sigma_{ABC}^2 + \text{cov share}_B)$

   $= 0.005 \times (383.13 + 541.762)$

   $= 4.624$

$R_{total} = R_{ABC} + R_B = 19.557 + 4.624 = 24.181$

Part b: 2.0 points

Solution 1

   Portfolio A has high coef var (8.62) compared to B, but A has lower covariance with ABC, so A is better in terms of diversification.

   RAROC: Using given profit/total C_i

   $\text{RAROC}_{ABC + A} = \frac{3.38}{22} = 15.4\%$ EVA = 3.38 - .15 x 22 = .08

   $\text{RAROC}_{ABC + B} = \frac{4.28}{25} = 17.12\%$ EVA = 4.28 - .15 x 25 = .53

   $\text{RAROC}_{ABC \, abne} = \frac{3.2}{20} = 16\%$

   Each total portfolio has RAROC > 15%, so adds value, profitable.

   Portfolio B is larger (greater TIV, more locations), so will add more to EVA b/c bigger.

   Rate increases on ABC:
   Stand-alone renewal risk load for ABC is .005 x 2538.69 = 12.69, similar to what renewal risk load is under ABC + A (12.82). Risk load for ABC much higher if B added (big rate increase).

   Considering elements above (correlation of new part w/current ABC, RAROC/EVA for scenarios and renewal risk load chance to ABC), I would suggest adding portfolio A due to added diversification, adequate RAROC and no big increase in risk load on renewal for current ABC portfolio.

Solution 2
Since renewal risk loads both ABC and each account needs their share of covariance

$\text{RAROC}_{A \& ABC} = \frac{Inc}{Risk\ Adj\ Capital}$ $\text{RAROC}_{B \& ABC} = \frac{4.28}{25}$

$$= \frac{3.38}{22} = .154 \qquad\qquad\qquad = .1712$$

- So both have RAROC > 15%, so they are ok there
  B is higher so that is a consideration as well
- Adding A yields a total risk load of 12.91, adding B yields a risk load of 24.18. The risk load of B is much higher, almost twice as much.
- Adding B requires capital to increase from 20 to 25, while A only increase to 22, 15% more capital, while its RAROC is only 11% higher. $\frac{.171}{.154} \leq 11\%$
- A has a higher coefficient of variation, so its losses have more variability. On the other hand, its losses are negatively correlated w/ABC, it has low losses when ABC high and vice versus
- TIV per location A 45/100 = .45   B 160/250 = .64
  So B has more expensive properties
- I would choose A because of the negatively correlated losses, less increase in risk capital, and lower overall risk load.

Solution 3
- The risk load size is much bigger for B than A (almost twice)
- Portfolio B is better diversified geographically which makes it attractive as a CAT would less likely devastate company
- Both $RAROC_A = \frac{3.38}{22}$ = 15.36% & $RAROC_B = \frac{4.28}{25}$ = 17.12% are adequate but B is higher and one event is a lower chance of making it fall below required RAROC whereas a large loss for A can cause RAROC to fall below 15%
- Portfolio A is less volatile, as the variance/standard deviation will be much lower
- Portfolio A will lower RAROC, $RAROC_{ABC} = \frac{3.2}{20}$ = 16%
- Should find out if insured values are same for each location in Portfolio, as opposed to say 159 in 1 location and 1 in 249 locations for B.

Solution 4
Points of considerations:
1) Expected risk-adj rate of return (RAROC) => B is better

ABC RAROC = 3.2 / 20 = 16%
ABC + A RAROC = 3.38 / 22 = 15.3636% ⎱ ABC + B is larger (better)
ABC +B RAROC = 4.28 / 25 = 17.12%

⇨ All are > 15% required RAROC
But ABC + B's RAROC is larger than ABC + A's RAROC
Therefore prefers add port B

2) Relative riskiness of each new portfolio. (CV) => B is better

ABC CV = 4.14
A CV = 8.62   B is smaller (better) more stable, hence less risky
B CV = 4.4

3) Risk load & price competitiveness => A is better

Risk load to the added portfolio per $ of total insured value in the new portfolio
For A = 0.0842 / 45 = 0.00137   A provides a more (better) competitive rate.
For B = 4.62446 / 160 = 0.02890

4) Effects of diversification on ABC's existing portfolio.   => A is better

For A.   All of its loss distribution Is at the opposite of ABC. (e.g. for event 1, ABC should incur a large loss of $350, but A would only experience a loss of $2. For event 3, ABC would incur a loss of $1 only, but A would get almost a total loss $40 (out of $45). This is also demonstrated by a decrease in ABC's CV from 4.14 to 4.03 w/the addition of A.

For B. Most of the loss magnitude of B is in the same direction as ABC's existing portfolio. (not adding diversifications). This is also demonstrated by an increase in ABCj's CV from 4.14 to 4.19.

->A provides a better hedge and diversifies the risk faced by ABC's existing portfolio

Therefore, overall I would vote for A because the benefit of diversification and rate competitiveness outweighs the previous 2 points and that the RAROC for A still is exceeding our target RAROC

EXAMINER'S REPORT

This question was generally challenging given the many calculations involved in part a) and synthesis required in part b). Candidates most often received close to full credit on part a) and only partial credit on part b).

Part a

- Candidates were expected to know the formulas necessary to calculate the covariance shares for ABC with A and be able to recognize the covariance shares results were given for ABC with B. Candidates were then expected to know how to translate the appropriate variances and covariance shares into risk loads.
- Candidates were expected to present their calculations in a logical and understandable sequence and arrive at the correct results in order to receive full credit.
- Common errors made by candidates included: not knowing how to split the covariance shares between ABC and A, not recognizing the additional portfolios as renewal additive, dividing the given risk load multiplier by the standard deviation prior to application, and simple calculation mistakes.

Part b

- Candidates were expected to recognize multiple considerations Reinsurance Company ABC would need to make when adding an additional portfolio.
- In order to receive full credit, candidates needed to fully describe these considerations which included presenting what the considerations were and why they were relevant. The

most common considerations mentioned included RAROC, relative riskiness, diversification and total insured value.

- The most common errors made by candidates included: only describing one consideration, doing a calculation (relative riskiness for example) and not including an explanation or justification as to its relevance/importance, and not comparing the computed RAROCs to the target or the current RAROCs.

QUESTION 23	
TOTAL POINT VALUE: 1.75	**LEARNING OBJECTIVE: D8**
SAMPLE ANSWERS	

Q23: 1.75 points

Solution 1

SWAP   (normal distr.)

99.9^{th} ->   Z = 3.0902

M_L = 4m v_f=4%

G_L= 6m   y = 12%   Gy = 15%

From $s = \frac{s - M_L}{G_L}$ -> $s = 3.0902(6m) + 4m = 22.5412m$

Safety constraint; $A = \frac{s - M_L}{Hy} = \frac{22.5412m - 4m}{1.12} = 16.555m$

var. constraint; $A = \frac{G_L}{G_y} = \frac{6m}{0.15} = 40m$

select max A from both constraint -> $40m$

therefore asset required = $40m$

Solution 2

Loss safety constraint   Variance constraint

$A \geq \frac{S_L - M_L}{1+y}$ $A \geq \frac{\sigma_L}{\sigma_y}$

$\geq \frac{22.5412M - 4M}{1.12}$ $A \geq \frac{6M}{.15} = 40,000,000$

$= 16,554,642$

$M_L + z(\sigma_L)$

S_L = 4M + 3.0902(6M) = 22.5412M

Choose the more restrictive constraint on assets

Assets must be greater than 40,000,000

EXAMINER'S REPORT

The candidates were expected to know how to use Kreps' swap method to calculate the assets a reinsurer needs to allocate to support writing a policy. Candidates generally performed well on this question, either getting full credit or having minor calculation error.

The most common errors made, aside from numerical calculation mistakes, were:
- Incorrect/omitted formula for the loss safety level, S
- Use of the risk free rate in the calculation of the assets for the loss safety constraint
- Calculating the risk loads for the loss safety/variance constraints, instead of the assets
- Adding the premium to the final allocated assets to give funds invested
- Selecting the loss safety constraint as dominant instead of the variance constraint

A small percentage of candidates appeared to be confused by the final sentence of the question, which asked "Using the swap method, calculate the assets required for the reinsurer to cover a loss at the 99.9th percentile." The swap method requires calculation of both the loss safety and investment variance constraints, and then using the higher of those assets. These candidates typically only calculated the loss safety constraint, or calculated both, and then indicated the question asked for the

loss safety constraint as the answer.

QUESTION 24	
TOTAL POINT VALUE: 3	**LEARNING OBJECTIVE: D.7**
SAMPLE ANSWERS	

Part a: 2 points

Solution 1

Year	Inflation Prem	Inflation adj Reserves	% Incur in Reserves	% Incur in Demand
1	100M	50M		
2	100M	52,500,000	5%	0%
3	100M	53,125,000	5%	0%

$= 102M / 1.02$ $57,352,050/1.02^2$ $=\frac{55125}{52500} - 1$ $=\frac{100}{100} - 1$

Required % change in surplus = Inflation + Δ Reserves + Δ Demand
$$= 2\% + 5\% + 0$$
$$= 7\%$$

Solution 2

increase demand for insurance: 2%

year 2: $\frac{1.02}{100} = 1.02$ year 3: $\frac{104.04}{1.02} = 1.02$

increase on reserve   7.1%

year 2: $\frac{53.550}{50} = 1.011$ year 3: $\frac{57.35205}{5355} = 1.011$

Since numbers are on a nominal basis, take into consideration inflation.

Increase demand for insurance:	2%-2% = 0%
Increase on reserve.	7.1% -2% =5.1%
Inflation	2%

Required change in surplus = sum of above = 0% + 5.1% +2% = 7.1%

Part b: 1 point

Solution 1

required rate of return = required change in surplus + stockholder dividend – paid in capital

$$= 7.1\ \% + \frac{1.5}{20} - \frac{1}{20} = 9.6\%$$

EXAMINER'S REPORT
Part a
Most common error was not adjusting insurance demand for inflation. Second-most common error was not adjusting reserves for insurance demand. Accepted whether discounted year 2 and year 3 to year 1, or inflated year 1 and year 2 to year 3. Accepted whether adjusted for inflation/insurance demand each year, or adjusted the answer after calculating nominal change.
Part b
The most common error was forgetting to show the formula. Full marks could be obtained even if the response from Part a) was not correct.