1. (6 points)

An investor considers the following information.

- Risk-free rate, $r_f = 2.5\%$
- Risky portfolio A: risk premium = 7.5% and standard deviation =35%
- Risky portfolio B: risk premium = 3.0% and standard deviation =20%

Assume the investor wishes to invest in one and only one of the following options:

I. 100% invested at risk-free rate
II. 25% invested at risk-free rate and 75% invested in risky portfolio A
III. 25% invested at risk-free rate and 75% invested in risky portfolio B

This investor has risk aversion, A = 2, and uses the following utility function to compare portfolios

$$U = E(r) - \frac{1}{2}A\sigma^2$$

For each option, 1000 simulations of potential returns are performed and partially reproduced below.

	risk-free	risk-free + A	risk-free + B
Simulation	I	II	III
1,000	2.5%	-50%	-42%
999	2.5%	-47%	-38%
998	2.5%	-45%	-36%
997	2.5%	-42%	-35%
996	2.5%	-40%	-34%
995	2.5%	-37%	-33%
994	2.5%	-35%	-32%
993	2.5%	-32%	-31%
992	2.5%	-30%	-31%
991	2.5%	-27%	-30%
990	2.5%	-27%	-30%
989	2.5%	-26%	-29%
988	2.5%	-26%	-29%
987	2.5%	-25%	-28%
986	2.5%	-25%	-28%
985	2.5%	-25%	-27%
984	2.5%	-24%	-27%
983	2.5%	-24%	-27%
982	2.5%	-23%	-26%
981	2.5%	-23%	-26%

a. (1.5 points)

Calculate the utility for each of the three options.

b. (1.5 points)

Construct a plot containing

i. the capital allocation line for risky portfolio B
ii. an indifference curve for this investor

Label relevant intersections and axes.

c. (1 point)

Calculate the 1 in 100 conditional tail expectation (CTE) for each of the three options.

d. (1 point)

Calculate the 1 in 100 value at risk (VaR) for each of the three options.

e. (1 point)

Briefly describe one difference and one commonality between VaR and CTE. Compare the performance of portfolios II and III under each measure.

SOLUTION GUIDE

a. BKM, pp. 171

$E(R_p) = r_f + y(R_i)$

I. Risk-free E(r) = 2.5%

II. E(r) = 2.5% + .75 (7.5%) = 8.125%

III. E(r) = 2.5% + .75 (3%) = 4.75%

$\sigma_p = y(\sigma_i)$

I. $\sigma_p = 0\%$

II. $\sigma_p = .75\ (35\%) = 26.25\%$

III. $\sigma_p = .75\ (20\%) = 15\%$

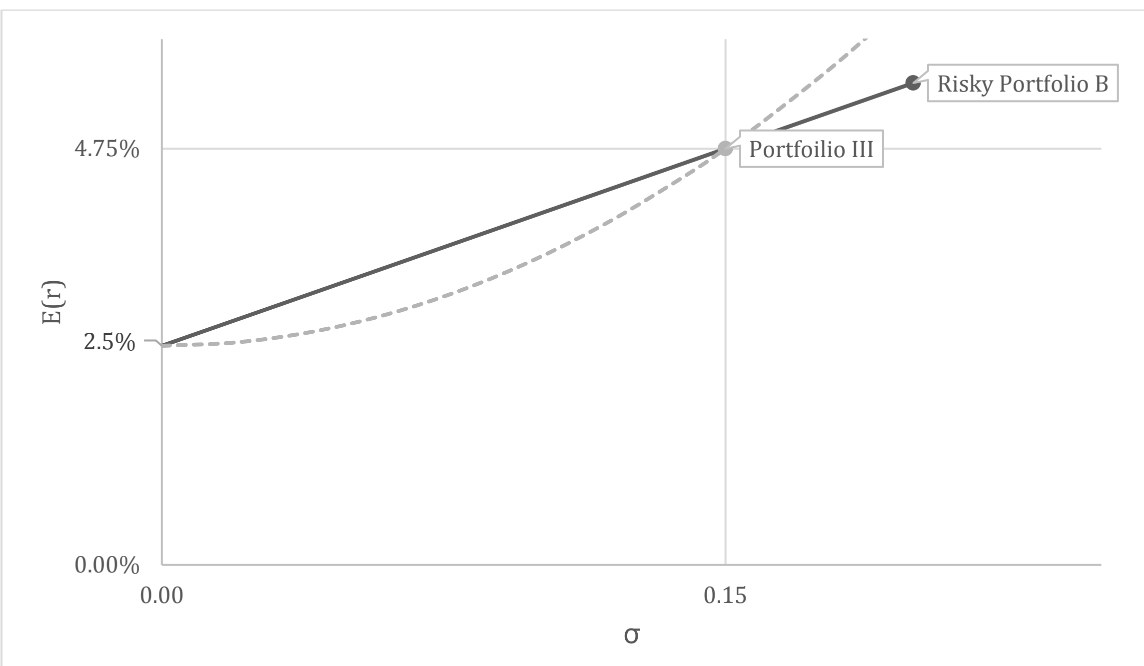
$U = E(r) - \frac{1}{2}A\sigma^2$

I. U = 2.5% - 0.5(2)(0^2) = 0.025

II. U = 8.125% - 0.5(2)(0.225^2) = 0.0123

III. U = 4.75% - 0.5(2)(0.15^2) = 0.025

b. BKM, p.173 & 179



c. Goldfarb, pp. 9-10

CTE = average (1% or 10 of 1000 lowest returns)

I. CTE = 2.5%

II. CTE = average (-50%, -47%, … , -30%, -27%) [simulations 1000:991] = -38.5%

III. CTE = average (-42%, -38%, …, -31%, -30%) = -34.2%

d. Goldfarb, pp. 8-9

VaR = return where there is 1% probability the return is at equal or worse than that return = simulation 991

I. VaR = 2.5%

II. VaR = -27%

III. VaR = -30%

Also would accept values from simulation 990

e. Goldfarb, pp. 9

Both measures: calculate a loss/capital/return amount based on a chosen threshold; primarily used to measure extremities in the distribution, also known as the "tail"; other relevant similarities

Difference: VaR reflects the value at a single percentile of the distribution whereas CTE represents the average loss for those losses that exceed the chosen percentile; other relevant differences

The 100% risk-free portfolio performs best under both CTE and VaR simply because it has no risk and therefore no downside risk. Of the other two options, Portfolio II performs better using VaR as a risk measure, whereas Portfolio III performs better using CTE.

2. (7 points)

An investor has $1,000,000 and would like to allocate these funds to a combination of risk-free and risky investments.

a. (2 points)

The investor can choose from among two stocks to construct his risky portfolio. Historical return, standard deviation, and correlation information from the past 50 years for these two stocks are presented in the below table, where R_i represents the excess return of stock *i*:

Stock *i*	$\overline{R_i}$	σ_i	$corr(R_i, R_j)$
A	.12	.20	.25
B	.04	.067	.25

Given the following information about the market:

- $E(R_m) = .06$
- $\sigma_m = .10$
- $r_f = .04$

i) (1 point)

Using the Markowitz procedure, construct the optimal complete portfolio for this investor, along with the expected return and standard deviation of this portfolio. Assume the investor follows a utility function with a risk aversion factor of A = 4.

The investor finds a third stock, Stock C, that can be included in his portfolio. Based on historical data from the past 50 years, the investor determines the following about Stock C:

Stock *i*	$\overline{R_i}$	σ_i	pair(i, j)	$corr(R_i, R_j)$
C	.09	.15	(A,C)	.75
			(B,C)	.25

ii) (0.5 point)

Will the Markowitz procedure produce the same optimal complete portfolio as the Single-Index Model for the three stocks and risk-free asset? Briefly explain why or why not using figures from the above example.

iii) (0.5 point)

Briefly discuss one reason why future returns may not be as extreme as they were in the historical period for the above stocks.

b. (2.75 points)

Upon further research, the investor feels confident that Stock A, an international oil stock, will achieve only a 10% return over the next year and believes there may be an arbitrage opportunity with the above risky assets. The investor does not believe the volatility of the stocks will differ from the historical values presented above.

i) (1.5 point)

Construct a potential arbitrage strategy that the investor can exploit based on the above information. Include the expected return and standard deviation of the strategy constructed.

ii) (0.75 point)

Briefly discuss three reasons why the above strategy may fail to achieve risk-free returns at a rate in excess of the risk-free rate.

iii) (0.5 point)

Use the existence of the above arbitrage opportunity as an argument in favor of or against the efficient market hypothesis.

c. (2.25 points)

The following is an example of a multifactor model, using factors describing risks:

$$E(r_i) - r_f = a_i + b_i\left[E(r_M) - r_f\right] + c_i E[SMB] + d_i E[HML] + e$$

where:

$E(r_M) - r_f$ - Market factor
SMB - Size factor (small firm return minus big firm return)
HML - Value factor (High book:market return minus low book:market return)

Factor	**Value**
SMB	2.5%
HML	2.0%

The investor calculates the risk premiums associated with the above multifactor model and derived the following results:

Stock	a_i	b_i	c_i	d_i
A	0%	1.5	1.5	1.0
B	1%	0.75	-0.75	-1.1
C	0%	0.8	0.8	0.6

i) (0.5 point)

Briefly describe two reasons why this multifactor model may outperform a single index model.

ii) (1.25 points)

Calculate the revised expected return of the portfolio calculated in a.i above, based on the above multifactor model

iii) (0.5 point)

Identify two additional potential risk factors that are not included in the multifactor model.

SOLUTION GUIDE

a.

i)

A. Calculate optimal risky portfolio weights:

$$w_A = \frac{E(R_A)\sigma_B^{\ 2} - E(R_B)\text{cov}(R_A, R_B)}{E(R_A)\sigma_B^{\ 2} + E(R_B)\sigma_A^{\ 2} - \left[E(R_A) + E(R_B)\right]\text{cov}(R_A, R_B)}$$

$$w_A = \frac{.12(.067^2) - (.04)(.2)(.067)(.25)}{.12(.067^2) + .04(.2^2) - (.12 + .04)(.2)(.067)(.25)} = .252$$

$$w_B = 1 - .252 = .748$$

B. Calculate the expected return and standard deviation of the optimal risky portfolio

$$E(R_p) = w_A E(R_A) + w_B E(R_B) = .252(.12) + .748(.04) = .06$$

$$E(r_p) = E(R_p) + r_f = .06 + .04 = .10$$

$$\sigma_p = \sqrt{w_A^{\ 2}\sigma_A^{\ 2} + w_B^{\ 2}\sigma_B^{\ 2} + 2w_A w_B \text{cov}(R_A, R_B)} = \sqrt{(.252^2)(.2^2) + (.748^2)(.067^2) + 2(.252)(.748)(.2)(.067)(.25)} = .078$$

C. Calculate optimal complete portfolio weight: $y = \frac{E(r_p) - r_f}{A\sigma_p^{\ 2}} = \frac{.10 - .04}{4(.078)^2} = 2.46$

D. Calculate optimal complete portfolio as well as expected return and standard **deviation**

Stock A: (.252) x (2.46) x \$1 million = \$621,163

Stock B: (.748) x (2.46) x \$1 million = \$1,840,136

Risk-free assets: (1-2.46) x \$1 million = \$-1,461,299

$$E(r_C) = yE(R_P) + r_f = 2.46(.06) + .04 = .188$$

$$\sigma_C = y\sigma_P = 2.46(.078) = .1924$$

ii) No. The single index model assumes the correlation between two securities can be purely explained by correlation with market returns, while the Markowitz procedure does not make this assumption. In the above example, stocks A and C are significantly more correlated under the single index model assumption than what the Markowitz procedure would use.

iii) Over time, the adjusted Beta approach assumes that returns and volatility of stock results will trend closer to the market figures. The above stocks show significant Beta values when compared to the market returns and it may be more appropriate to temper these values in future financial projections.

b.

Note that there are multiple correct answers to this question. The following represents a sample solution.

i) Calculate two pairs of stocks with different risk premiums relative to their β's

$$\beta_A = \sigma_A / \sigma_M = .2/.1 = 2$$
$$\beta_B = .067/.1 = .67$$
$$E(R_A)/\beta_A = .06/2 = .03$$
$$E(R_B)/\beta_B = .04/.67 = .06$$

An arbitrage opportunity exists by buying stock B and selling stock A. A zero Beta portfolio is:

$$w_A = \frac{\beta_B}{\beta_B - \beta_A} = .67/(.67-2) = -.5038$$
$$w_B = 1-(-.5038) = 1.5038$$
$$E(r_P) = .04 + 1.5038(.04) - .5038(.06) = .07$$
$$\beta_p = 0$$
$$\sigma_p = 0$$

If the investor purchases \$1.5038m of Stock B and shorts \$0.5038m of Stock A, his expected return will be \$70,000 with a standard deviation of \$0.

ii) 1. Implementation costs - Transaction costs associated with buying and buying and selling stocks may reduce or eliminate the arbitrage potential. Alternatively, stock A is an international stock, so liquidity concerns may make it difficult to complete the arbitrage transaction, especially if the market is thinly traded
2. Fundamental risk – You may incur short term losses before mispricing corrects itself, and the losses may get worse before they get better
3. The 10% return estimate may be incorrect

iii) The efficient market hypothesis states that stock prices already reflect all available information. The existence of the above arbitrage opportunity relies upon the assumption that some of the risk for stock A is not explained by the market. If the market were truly efficient, the existence of the above opportunity would disappear almost instantaneously.

c.

i) 1. The multifactor model may explain more of the systematic risk inherent in the portfolio than the single index model. For example, company size may be an indicator of systematic risk.
2. Investors may irrationally prefer certain companies over other companies even if they have the same riskiness. For example, investors may prefer large companies over small companies even if they are not less risky.

ii) Calculate the expected return for each stock

$$E(r_a) = r_f + \alpha + b_a R_M + c_a SMB + d_a HML = .04 + 0 + 1.5(.06) + 1.5(.025) + 1.0(.02) = 18.75\%$$
$$E(r_b) = .04 + .01 + .75(.06) - .75(.025) - 1.1(.02) = 5.43\%$$

Calculate the expected return for the portfolio

$$w'_a = yw_a = (2.46)(.252) = .6212$$
$$w'_b = yw_b = (2.46)(.748) = 1.8401$$
$$w'_{rf} = 1 - 2.46 = -1.4613$$
$$E(r_p) = w_a E(r_a) + w_b E(r_b) + w_{rf} E(r_{rf})$$
$$= .6212(.1875) + 1.8401(.0543) - 1.4613(.04)$$
$$= 15.78\%$$

iii) 1. Spread of returns between long-term government bonds and t-bills

2. Spread of returns between high and low quality long-term corporate bonds

3. (7.25 points)

A publicly-traded multiline property and casualty insurer is conducting a review of the performance of its lines of business. The following information pertains to the insurer and its review:

- The insurer uses the Fama-French Three-Factor model to estimate its required cost of capital.
- The insurer holds capital based upon the Value at Risk (VaR) at the 99th percentile.
- The full cost of capital for the insurer is allocated to each of its underwriting lines of business using the "Capital Allocation by Percentile Layer" approach by Bodoff
- The risk-free rate of return is 2%
- An empirical analysis of stock market returns results in the following estimates:

Factor	Risk Premium	Company Beta
Market Index	5.0 %	1.2
Small Minus Big	3.5 %	-0.3
High Minus Low	4.5 %	0.8

- The insurer writes three lines of business with all policies effective on January 1 of each year:

Line of Business:	A	B	C
Premium ($millions)	250	1,000	350
Operating and Underwriting Expense (% of premium)	25%	30%	32%
Expected Client Retention	75%	90%	95%

- Premium is collected and expenses paid on January 1 of each year. Losses are paid on December 31 of each year.
- The insurer uses the following set of loss scenarios to approximate the distribution of total annual losses:

		Expected Claims ($millions)			
Scenario	Probability	A	B	C	Total
1	0.600	175	520	80	775
2	0.250	180	580	100	860
3	0.100	185	750	250	1,185
4	0.040	190	1,500	1,200	2,890
5	0.010	250	2,000	2,500	4,750
Expected Value		178.6	612.0	171.0	961.6

- The insurer operates in a jurisdiction that is regulated using the following benchmarks:
    - Risk ratio (premium-to-surplus) of 3:1
    - Allowable return on equity of 15%

a. (0.5 point)

   Calculate the insurer's cost of capital.

b. (3 points)

   Calculate the capital allocated to each of the lines of business.

c. (1.5 points)

   Calculate the insurer's total economic value, and determine which of the insurer's lines of business has the greatest franchise value.

d. (0.75 point)

   For each of the insurer's lines of business, evaluate the expected Risk-Adjusted Return on Capital against the insurer's cost of capital.

e. (1.5 points)

   Identify one concern each of the following parties might express regarding the profitability of the insurer's lines of business, and provide a recommendation for the insurer to address each concern.

   Briefly explain how the recommendation addresses each party's concern:

   (i) the insurer's shareholders
   (ii) the insurer's regulator

SOLUTION GUIDE

a. {BKM chapter 10, p340 (10^{th} ed) }

Cost of capital is the expected stock return. Using Fama-French Three Factor (FF3F) model:

$$E(r) = r_f + \beta_M R_M + \beta_{SMB} R_{SMB} + \beta_{HML} R_{HML}$$

$$= 2\% + 1.2 * 5\% + -0.3 * 3.5\% + 0.8 * 4.5\% = 10.55\%$$

b.

First, determine the total amount of capital that the insurer is holding. VaR_{99} corresponds to scenario #4. Therefore, the total capital, including contributions from premium (net of expenses) must be equal to the present value of the loss in scenario 4.

So S = L/(1+y) – P(1-e) {where e = expenses as % of premium}

[premium offset is mentioned in section 7.2.1 on page 21 of Bodoff.

The formula above is an extension of Bodoff and Panning (discounting loss)]

= 2,890 / (1.02) – [250 * (1-0.25) + 1000 * (1-0.3) + 350 * (1-0.32)] = 1707.83

Next need to allocate this capital by percentage layer.

Since the company is holding capital (and premium) to cover *up* to the VaR(99%) level, the cost of capital will be allocated up to the level of scenario 4.

Thus, we have four percentile layers of "capital" for the allocation procedure: \$775M x 0M, \$85M x 775M, \$325M x 860M, and \$1705M x 1185M.

The first layer will be allocated to all scenarios proportionally to their probabilities. Similarly, the second layer will be allocated to scenarios 2 through 5, and so on:

Scenario	Prob	Allocation of layer 1	Allocation of layer 2	Allocation of layer 3	Allocation of layer 4	Total Allocation
1	.60	775 x .6 = 465	NA	NA	NA	465
2	.25	775 x .25 = 193.75	85 x .25 / .4 = 53.125	NA	NA	246.875
3	.10	775 x .10 = 77.5	21.25	325 x .1/.15 = 216.667	NA	315.4167
4	.04	775 x .04 = 31	8.5	86.667	1705 x .04/.05 = 1364	1490.1667
5	.01	7.75	2.125	21.667	341	372.5417
Total		775	85	325	1705	2890

Now we have to allocate these back to each line of business, according to the proportion of loss in each scenario:

Scenario	Allocation	A	B	C	Total
1	465	465 * 175 / 775 = 105	312	48	465
2	246.875	246.875 * 180 / 860 = 51.67	166.50	28.71	246.875
3	315.4167	49.24	199.63	66.54	315.4167
4	1490.1667	97.97	773.44	618.75	1490.1667
5	372.5417	19.61	156.86	196.07	372.5417
Total	**2890**	**323.49**	**1608.43**	**958.08**	**2890**

Finally, as we did above for the entire firm, we need to set allocated capital equal to the present value of the loss amounts above, "offset" for the contribution of premium (less expenses): {Bodoff, p21}

$S_i = L_i/(1+y) – P_i(1-e_i)$

Line A: S_A= 323.49 / (1.02) – [250 * (1-0.25)] = 129.65

Line B: S_B = 1608.43 / (1.02) – [1000 * (1-0.3)] = 876.89

Line C: S_C = 958.08 / (1.02) – [350 * (1-0.32)] = 701.29

Note: the allocation calculations may be done on an undiscounted basis, discounting at the end, as was done here, or the values could be discounted initially and all calculations done on a discounted basis. It should not affect the answers.

c.

Current economic value = S + P(1 – e) – L/(1+y) {Panning, p6}

CEV = 1707.83 + [250 * (1-0.25) + 1000 * (1-0.3) + 350 * (1-0.32)]– 961.6/(1.02) = 1890.6

Franchise value = F = [P-E-L/(1+y)] * d/(1-d), where d = cr/(1+y) {Panning, p7}

Then:

d_A = .735 F_A = [250(1-.25) – 178.6/1.02] *(.735/.265) = 34.4

d_B = .882 F_B = 750.0

d_C = .931 F_C = 954.8

Therefore, line of Business C has the highest franchise value.

Total F = 34.4 + 750.0 + 954.8 = 1739.2

Total economic value = current economic value + franchise value {Panning, p7}

TEV = CEV + F = 1890.6 + 1739.2 = 3629.8

d.

Expected RAROC = Expected Income / Risk-Adjusted Capital {Goldfarb p4}

$= [P_i(1-e_i) * (1+y) - L_i] / S_i$

A: [250* (1-0.25) * 1.02 – 178.6] / 129.65 = 9.76% - this is **lower** than the cost of capital (10.55%)

B: [1000* (1-0.30) * 1.02 – 612] / 876.89 = 11.63% - **higher** than the cost of capital

C: [350 * (1-0.32) * 1.02– 171.00] / 701.29 = 10.23% - **lower** than the cost of capital

Notes:

1. Some candidates may discount the entire RAROC evaluation to time t=0. As long as the approach is consistent and valid, full credit would be given. Goldfarb's main example is done at time t=1 using actual results, while this question is asking candidates to project the expected RAROC from time t=0. Using expected t=1 results instead of actuals is sufficient to address this difference, although some candidates may feel a need to try to bring the RAROC back to time t=0.
2. Goldfarb identifies several issues with the comparison asked for in this question {p. 43} but concedes, "One acceptable compromise is to recognize that models such as CAPM or the Fama-French 3-Factor Model are reasonable means to quantify shareholders' target return on the firm's total capital (e.g. GAAP book value). {p. 44}

e.

Concerns:

(i) Shareholders could express concern that Lines A and C have a RAROC that is lower than the FF3F cost of capital. {McClenahan p117}

(ii) Regulators could express concern that the "rate of return" measure for Line C using the benchmark risk ratio is 60%, and for line B it is 30%, both much higher than the benchmark level of 15%. {McClenahan p121}

Note: Other valid concerns are possible.

Recommendations:

- One recommendation to address the concerns of shareholders could be to increase the premium for lines A and C so that the expected returns meet the cost of capital. However, this could meet with regulatory resistance, since line C already has an expected rate of return on premium higher than the benchmark allowed, and line A is right at the benchmark.

- One recommendation to address the concerns of regulators could be to lower premiums for lines B and C to meet the benchmark returns. However, this would require the firm to hold even more capital, which would decrease even further the RAROC for these lines, pushing their expected returns below the cost of capital – especially line C.

- The insurer could exit unprofitable line C. This could address the concerns of both shareholders and regulators.

- One recommendation that might address both of these concerns is to purchase reinsurance for Line C. It could reduce the expected loss at the higher layers, reducing the need to hold as much capital, and also reducing the capital allocated to this line. By reducing the capital held and allocated to this line, it could both increase the RAROC and mitigate the need to price this line to an expected combined ratio of 80%, thereby addressing the concerns of both shareholders and regulators. (This would depend heavily on the pricing and structure of the reinsurance agreement and potentially on other existing reinsurance agreements, i.e. the leveraging of the capital reduction versus the cost of the reinsurance.)

Note: Many valid recommendations are possible. The ones listed here are but a few examples.