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How to Create a Market Cycle

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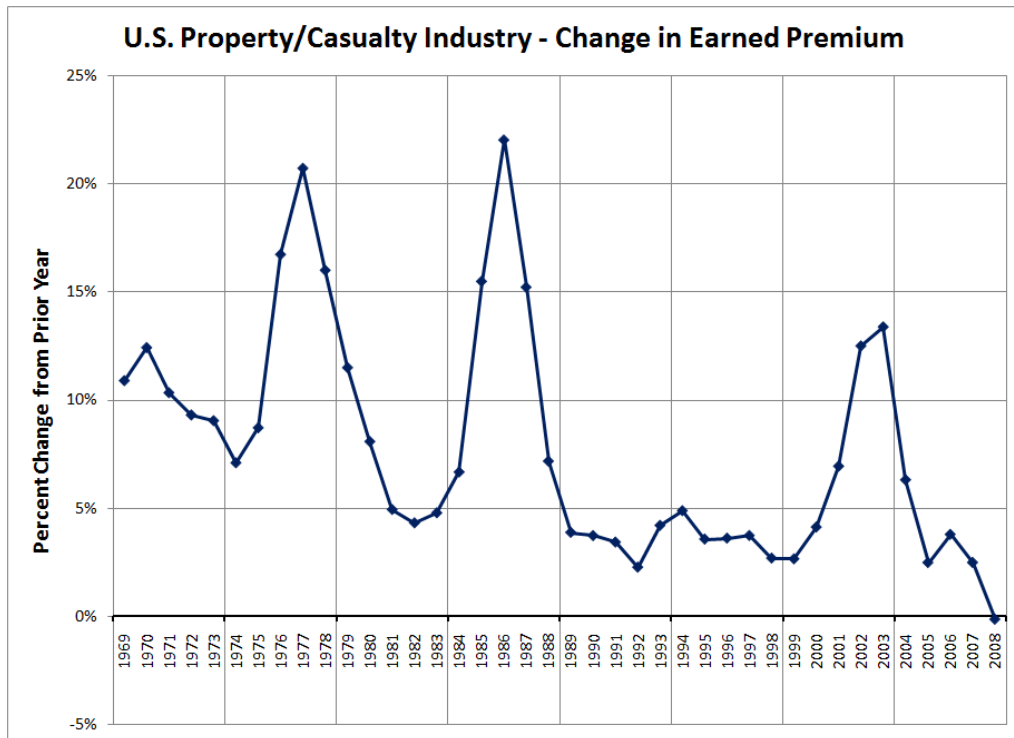
Abstract:

There has been much discussion about the possible causes of the market cycle in the Property & Casualty insurance industry but little consensus. This paper will offer a simplified behavioral model, showing how a market cycle can be created assuming naïve rules for reserving and pricing business. The model can be interpreted as a linear difference equation, which naturally gives rise to a cyclical pattern. This model is shown to be consistent with published financial results in several ways, and offers a testable leading indicator of future market turns.

Keywords: Market cycle, difference equation, time-series

1. INTRODUCTION

The market cycle in the Property & Casualty industry is an oscillating movement in profitability over time. The period (length in time) is irregular but recurs approximately each twelve to fifteen years. The cycle can be observed by looking at the change in industry earned premium.



The history shows rapid premium growth, driven by rate increases, in periods 1976-1978,

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1985-1987 and 2001-2004. These periods of rapid premium growth are informally known as “hard market” years. In the intervening years, premium is generally not keeping pace with inflation and so rate adequacy deteriorates; this is known as a “soft market.” The exact causes for this cyclical movement are not fully understood. The insurance cycles do not correspond to cycles in the larger economy as measured by GDP or interest rate movement. More surprisingly, the insurance cycle does not appear directly related to the occurrence of natural catastrophes.

The present paper will outline a model of simple behavior that is sufficient to create a cycle similar to that actually observed. The simplified model is meant only as a baseline to show the implications of that behavior, whereas a fully realistic model would need to incorporate many more factors.

1.1 Research Context

This paper makes assumptions about pricing and reserving behavior that can be defined in terms of recursive equations, technically known as linear difference equations. The mathematics can then explain the necessary consequences of the behavioral rules.

The behaviors described follow closely with what was given by Mango and Venter in “An Introduction to Insurer Operational Risk.”¹ They describe this behavior in terms of an agent-based model (ABM) in which numerical results follow from defining the rules that govern the behavior of the “agents” (active participants) in the market. The mathematical description of that behavior is made explicit in the present paper. The mathematics behind solving linear difference equations can be found in a number of standard time-series texts².

1.2 Objective

Our objective is to provide a simplified explanation of insurance company management behavior that is sufficient to create a market cycle. The explanation is sufficient in the sense that if some very limited assumptions are met then a cycle is created. We are not concluding that no other factors are involved or that these are the only factors that could produce the market cycle.

¹ Chapter 4 of Enterprise Risk Analysis

² See chapter 1 of Enders.

2. BACKGROUND AND METHODS

In this section we will outline the key assumptions in our model of the market cycle: first in business terms and then giving a more mathematical description.

2.1 Assumptions about Market Behavior

We will show an example of how a market cycle can be generated based on a few simple – even naïve – behavioral rules. These rules are derived from three key assumptions.

1. “True” expected losses by year follow an exponential growth pattern; in the numerical examples, the actual loss will be exactly equal to the expected loss.
2. Reserving is performed using a Bornhuetter-Ferguson (BF) method, using the same 65% permissible loss ratio (PLR) as the complement every year.
3. Pricing is based on the latest three years of calendar year booked loss ratios.

The reserving and pricing behavior described in these assumptions implies that the reserving actuaries believe that pricing is done accurately, and that the pricing actuaries believe that reserving is done accurately.

It is worth discussing these assumptions in more detail.

The first assumption is that losses follow a deterministic exponential growth pattern.

$$L_j = L_{j-1} \cdot (1 + g) \quad \forall j$$

Where L_j is the actual loss in year j

g is an annual growth rate

For the initial version of this model, losses are not treated as random variables or include any stochastic elements. This is because we wish to show that the stochastic element is not needed in order to create or perpetuate a cycle. This deterministic assumption produces a very smooth result; we can later add a stochastic element to produce variations in the pattern making it look more realistic to the eye.

We are also holding all parameters of the model constant over time. A more complex

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model would recognize other factors that change over time and therefore change the period (length) and/or amplitude of the cycle. Some of these other factors could be:

- Economic changes to inflation, interest rates or GDP; which affect the length of the payment pattern, the choice of PLR, and the competitive environment
- Random “shock” loss event such as catastrophes
- Changes to legal and regulatory environment
- Competitive pressures such as new entrants to the market (capital markets, health insurers, etc) or globalization

The second assumption is that reserving is performed using a Bornhuetter-Ferguson (BF) method, assuming a constant permissible loss ratio (PLR).

For this calculation, we need to define a payment pattern

$$\{\beta_i\}_{i=1}^{\infty} = \{\beta_1, \beta_2, \beta_3, \dots\} \text{ such that } \sum_{i=1}^{\infty} \beta_i = 1$$

We can afford to interpret this pattern as either paid or reported (“case incurred”) percents. Mathematically, this pattern is used to transition from the initial expected loss to the true ultimate loss by year. The pattern is best thought of as the time to recognize the true ultimate loss.

The estimated loss for year j as of development (or payment) period k can be denoted $\hat{L}_j^{(k)}$ and written as a weighted average of the “true” ultimate loss L_j and the premium times the permissible loss ratio.

$$\hat{L}_j^{(k)} = L_j \cdot \left(\sum_{i=1}^k \beta_i \right) + Prem_j \cdot PLR \cdot \left(1 - \sum_{i=1}^k \beta_i \right) \tag{1.01}$$

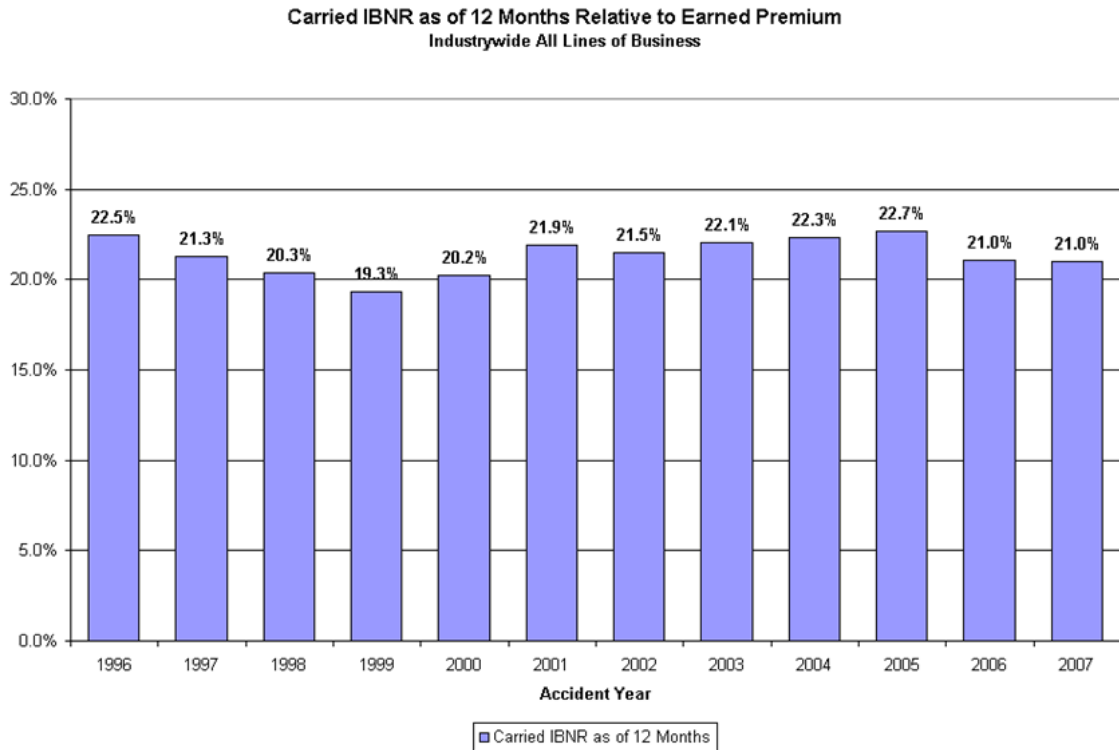
The interpretation of the pattern as a speed-to-recognize the true loss means that even short-tailed lines, such as commercial property, may have a long cycle. In property lines, some losses can be dismissed as “outliers” and therefore slow down an insurer’s recognition of the actual level of rate adequacy.

Finally, it should be noted that the assumption of reserving based on a constant PLR is

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very consistent with actual financial results consolidated at the industry level.

The following graph shows the amount of IBNR carried for each accident year as of the end of that accident year (evaluated as of 12 months). For the industry in total, these numbers are nearly constant by year.



The third assumption is that the premium for each year is set based on recent calendar year incurred losses (CYIL).

$$Prem_j = \frac{1}{n} \cdot \sum_{k=1}^n \frac{CYIL_{j-k} \cdot (1 + g)^k}{PLR} \tag{1.02}$$

This assumption may seem strange to a pricing actuary who would rarely, if ever, use calendar year experience in a rate filing³. Here it is best to remember that rate adequacy

³ As Felblum (2001) says, “Actuaries indicate rates, but the market sets prices.”

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refers to premium actually collected relative to expected loss; it is not simply the change in manual rates.

Many factors besides filed manual rates affect the actual premium and expected loss.

- The indicated rate level may not be approved by a state insurance department.
- Schedule and discretionary modifications to base rates may be market-driven.
- Enforcement of classification assignments may change under market pressure. For example, rates may be assigned based on “dominant” class for a risk or performed at a more detailed level. The use of the “not otherwise classified” classes may also change over time.
- Policy terms and conditions (exclusions, additional named insureds, etc) can expand or restrict coverage without a change in charged premium.
- Enforcement of contract provisions such as auditing of exposures can change over time.
- Manuscripted policies, excess & surplus lines, and reinsurance are generally not subject to filed rates.

The factors that change the rate adequacy from one year to the next are therefore not necessarily tied to actuarial indications. We are assuming that these factors are driven by a reaction to the most recent calendar year booked results.

One reason to make this assumption is that the suppliers of capital to the insurance industry (stockholders) are not always insurance company management with a full understanding of the distinction between accident year and calendar year results. Investors in insurance stocks may only review the calendar year combined ratios for the latest year and immediate prior year.

Likewise, regulators have an interest in making sure that the policyholders in their state are not charged excessive rates. Rates may be viewed as excessive if current financial results for the insurance industry are highly profitable – even if those results represent calendar year experience rather than current rate-level experience.

In assumption #2, we said that the industry’s use of the Bornhuetter-Ferguson reserving method introduces a lag in the recognition of ultimate losses. In assumption #3, we are

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saying that the industry also fails to recognize this lag. These two assumptions contribute to what a recent paper described as the “industry delusion factor⁴.”

Having defined these three assumptions, it is important to note that they apply at an overall industry level. We are not saying that each and every insurance company behaves in exactly this way. We are, so to speak, watching the movement of a flock of birds and not the movement of individual birds.

2.2 Mathematical Description

The behaviors described above are easily translated into mathematical language. We will provide the basic description now and segregate the more rigorous theory to Appendix A. For our actual analysis, it is sufficient to define the basic formulas to be used; the results will be numerical and graphical implementation of these formulas.

We begin by defining the calendar year paid and incurred loss amounts, denoted CYPL and CYIL. The CYPL represents payments made during a given year on the current and all prior years.

$$CYPL_j = \sum_{i=1}^{\infty} L_{j+1-i} \cdot \beta_i \tag{1.03}$$

Given our assumption that the ultimate losses follow a deterministic exponential growth, we can alternately write the formula for CYPL as below. It should be clear that the CYPL amounts grow with the same rate g as the ultimate losses and do not exhibit a cyclical pattern.

$$CYPL_j = L_{j-1} \cdot \sum_{i=1}^{\infty} (1 + g)^{-i} \cdot \beta_i = CYPL_{j-1} \cdot (1 + g) \tag{1.04}$$

The calendar year incurred loss, CYIL, is defined as the calendar year paid loss plus the change in the total reserve. The total reserve, in turn, is assumed to be calculated from the BF method applied to each accident year.

⁴ Underwood & Zhu (2009)

$$CYIL_j = CYPL_j + PLR \cdot Prem_j - PLR \cdot \sum_{i=1}^{\infty} Prem_{j+1-i} \cdot \beta_i \quad (1.05)$$

The CYIL amounts are dependent upon a recursive relationship with the historical premium amounts. This can introduce a cyclical pattern so that, in general,

$$CYIL_j \neq CYIL_{j-1} \cdot (1 + g).$$

The current year CYIL can alternatively be written as a recursive relationship with prior year CYIL amounts. This is known mathematically as a linear difference equation.

$$CYIL_j - L_j = \frac{1}{n} \cdot \sum_{k=1}^n \left\{ (CYIL_{j-k} - L_{j-k}) - \sum_{i=1}^{\infty} (CYIL_{j+1-i-k} - L_{j+1-i-k}) \cdot \beta_i \right\} \cdot (1 + g)^k \quad (1.06)$$

The “solution” to a linear difference equation is to write $CYIL_j$ non-recursively as a function of time j , not including the prior values of $CYIL_{j-i}$. The solutions are not easy to find but generally include a combination of exponential and trigonometric functions.

In a most basic case, we can set $n = 1$, $g = 0$ and $\{\beta_i\} = \{0, 1, 0, 0, 0, \dots\}$. This assumes that all payments are made in the second development year, and only one calendar year is used in pricing. This sets up the recursive relationship⁵:

$$(CYIL_j - L_j) = (CYIL_{j-1} - L_{j-1}) - (CYIL_{j-2} - L_{j-2})$$

The “solution” to this difference equation is a trigonometric formula with any arbitrary constants a and b .

$$(CYIL_j - L_j) = a \cdot \cos\left(j \cdot \frac{2\pi}{6} + b\right)$$

This shows that a cycle that repeats at six-year intervals is the exact solution generated

⁵ With addition of an error term, this is equivalent to a second order autoregressive process, AR(2), as proposed by Venezian (1985).

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from a payment pattern with a mass point in the second year. In other words, the pattern of calendar year incurred losses is not just similar to a cyclical pattern – it is exactly a cyclical pattern. Appendix A gives more general conditions for stable cycles to be created with this model.

More realistic input values for payment patterns, growth rates (g) and number of years to review in pricing (n) produce more complicated patterns. It is useful to understand the mathematics behind difference equations in order to gain insight into the cycle behavior, but it is not necessary in order to generate results from the model. For a reasonable range of input values, we can generate a stable cycle over a forty or fifty year time horizon.

2.3 Implied Relationship between Accident and Calendar Year Results

Before proceeding to calculating values from this model, it is useful to show one immediate implication of the assumptions we have made. That is the relationship between the “true” ultimate underwriting profit (or loss) for an accident year and the booked underwriting profit (or loss) for the calendar year.

The “true” ultimate underwriting profit is the amount by which the portion of the premium available to pay losses exceeds the actual ultimate loss. The mathematical definition is very concise.

$$ProfitAY_j = PLR \cdot Prem_j - L_j \quad (1.07)$$

For the calendar year booked underwriting profit, we compare the portion of the premium available to pay losses to the CYIL amount.

$$ProfitCY_j = PLR \cdot Prem_j - CYIL_j \quad (1.08)$$

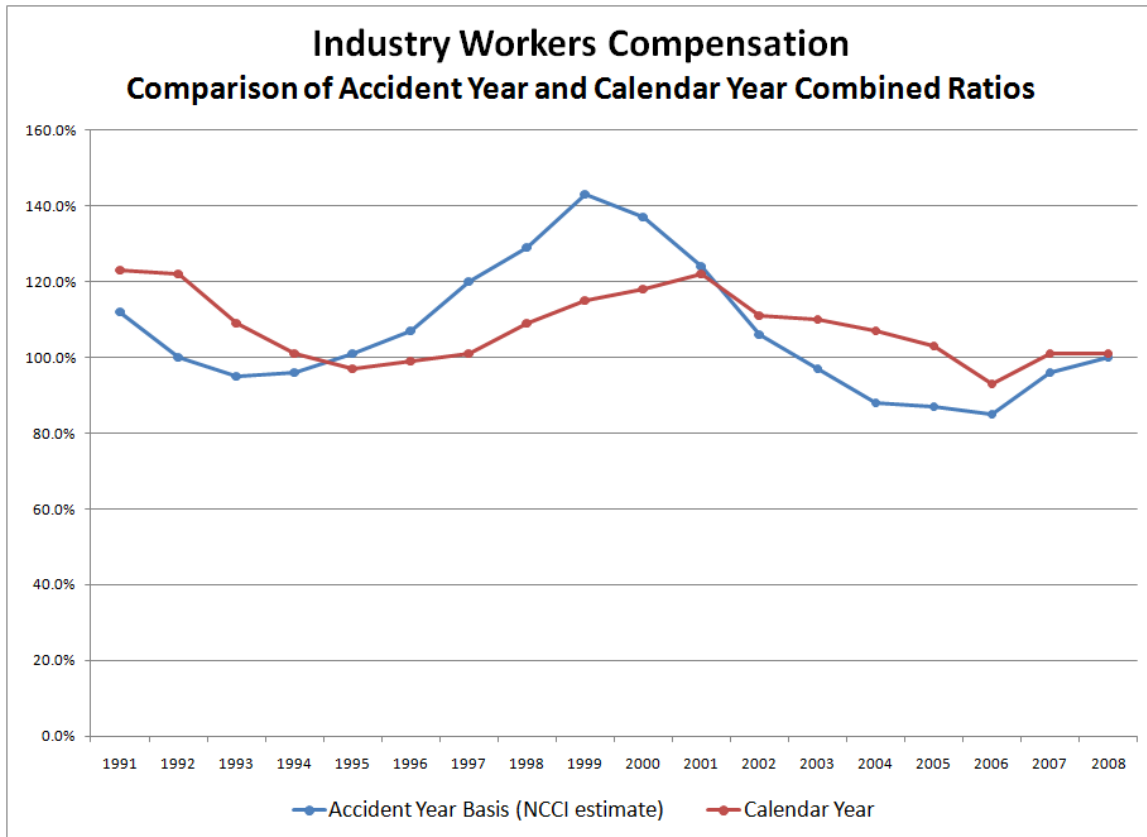
If we substitute in the formula for CYIL (formula 1.05), this is rewritten as below.

$$ProfitCY_j = \sum_{i=1}^{\infty} (PLR \cdot Prem_{j+1-i} - L_j) \cdot \beta_i = \sum_{i=1}^{\infty} ProfitAY_{j+1-i} \cdot \beta_i \quad (1.07)$$

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What this means is that the calendar year booked profit is equivalent to a rolling average of the accident year profits, using the payment pattern as weights in the average.

As a practical matter, this implies that the calendar year experience will show a smaller cycle magnitude than the accident year pattern, and be lagged by approximately the same amount as the average payment (or recognition) lag. The graph below shows this relationship for the Workers Compensation line of business.⁶



3. RESULTS AND DISCUSSION

We now proceed to the implementation of the model.

In order to calculate numbers from the proposed cycle model, we need estimates for the

⁶ See the State-of-the-Line reports published annually by the National Council on Compensation Insurance (NCCI) www.ncci.com. The graph take numbers from their 2001 and 2009 reports.

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key inputs:

Let $n = 3$ the number of calendar years included in the pricing review

Let $g = .02$ as the annual growth rate of industry losses

Let $PLR = 65\%$ permissible loss ratio

Let $\{\beta_i\}_{i=1}^{\infty}$ be estimated as some combination of industry paid and incurred patterns

Of the three key inputs, the pattern is the most difficult to select. As noted above, it is not clear that the BF method would be applied on a paid or incurred basis. Even if it is applied based on incurred losses, it is still possible that case reserve adequacy changes with the market cycle and so the correct answer should be somewhere between the two patterns.

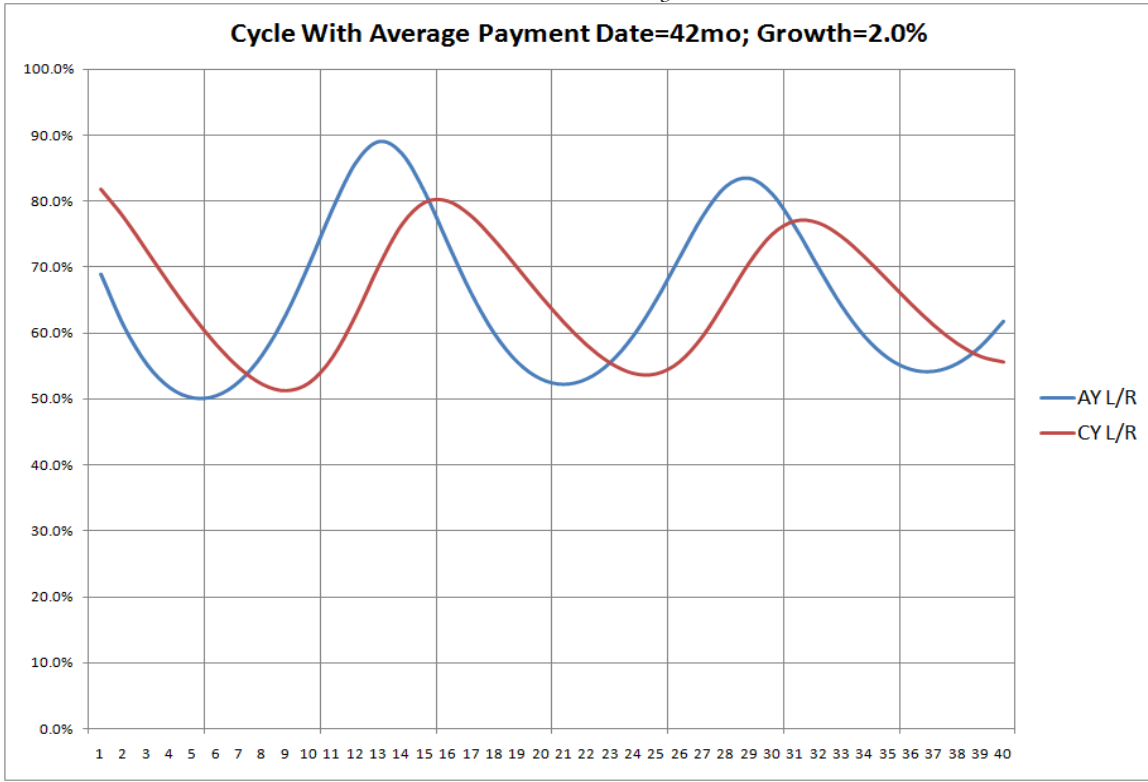
We will also want to select a pattern of finite length so that it can be incorporated into a numerical calculation.

Given these considerations, we will use in our examples a ten-year pattern with average payment date of forty-two months.

Year of Payment	Incremental % Paid
10	0.27%
9	0.74%
8	2.17%
7	5.28%
6	10.69%
5	17.64%
4	23.25%
3	23.18%
2	14.76%
1	2.02%

These assumptions produce a cycle of accident year and calendar year loss ratios as follows.

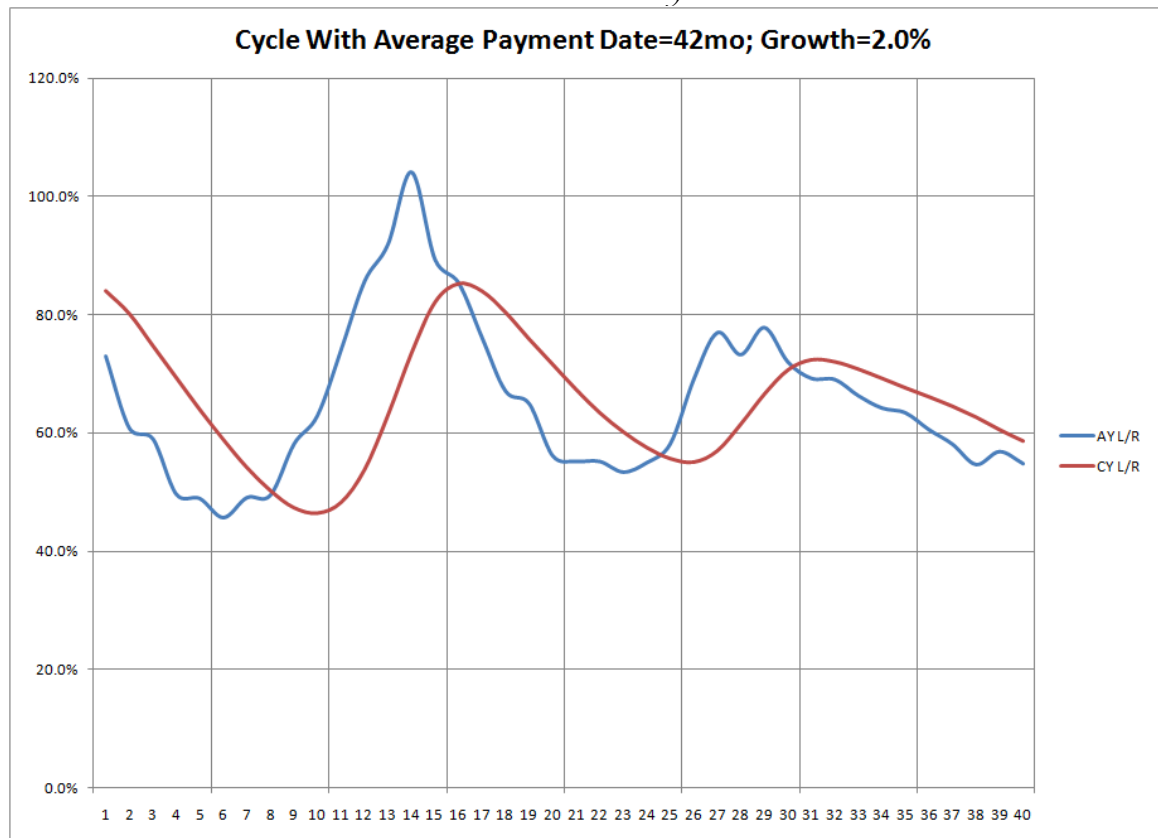
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The market cycle produced has a period (length for the pattern to be repeated) of about fifteen years. This is reasonable compared to the actual industry periods.

The smoothness and regularity of these curves is due to the deterministic loss assumption. If we replace the sequence of expected losses with values simulated from a random walk model, then a more irregular cycle is generated – perhaps appearing more realistic to the reader with experience in the insurance industry.

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The usefulness of this model is in the question of forecasting the movement in the insurance cycle. As we have seen above, the calendar year result is a lagging indicator of profitability – useful for showing where the market was a few years prior. We would like to find something that is more of a leading indicator.

In the idealized cycle (with no variability), all values repeat at regular intervals so there is no precise distinction between leading and lagging indicators. However, we can find some effects that are shifted slightly to the right or left of the accident year cycle. Those that are shifted to the left are leading indicators. Since there are many factors that in reality change the length or duration of a single cycle, such leading indicators are still needed.

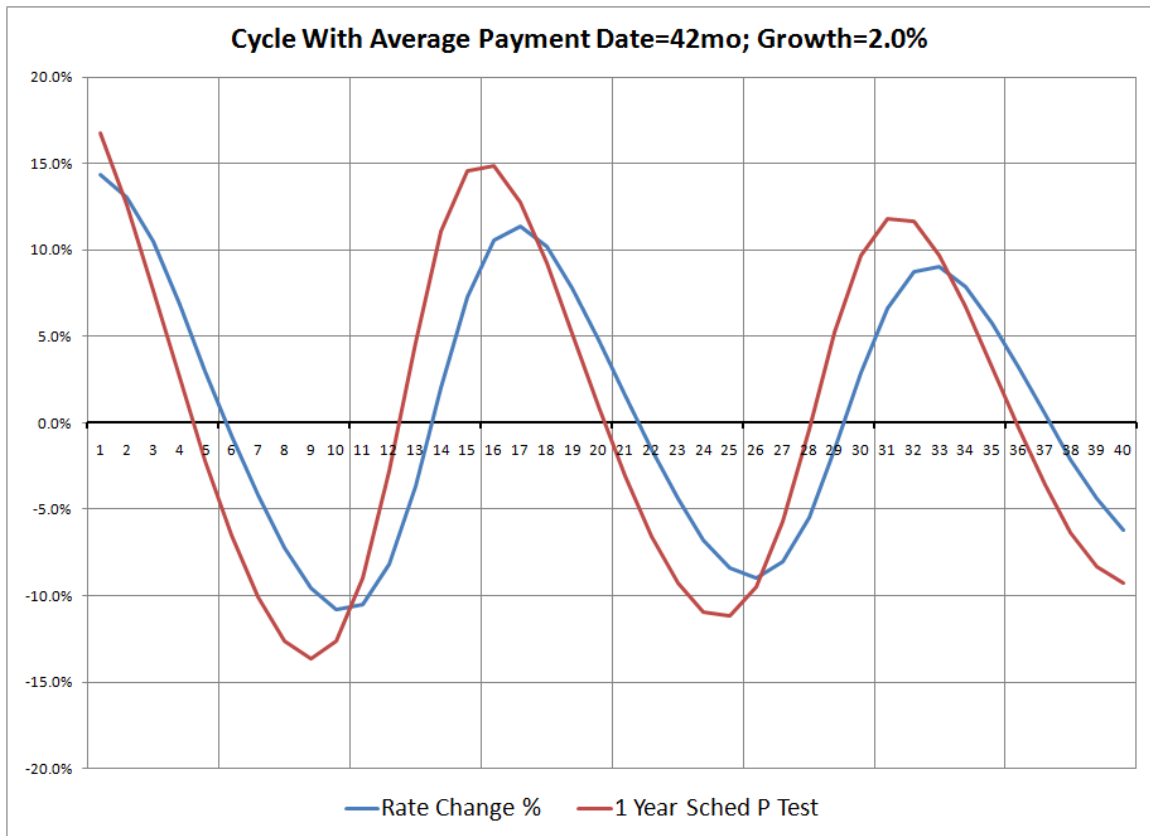
One possibility is the one-year development test available in the statutory Schedule P. This amount is the change in the estimated ultimate losses on years prior to the current accident year. This can be interpreted as a change in relative reserve adequacy⁷. In the graph

⁷ The use of this statistic in monitoring market cycle was first suggested by Todd Bault.

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below, this statistic (as a percent of premium) is graphed along with rate change, which is not directly observable in financial reporting.

The Schedule P test appears to be a good leading indicator of rate change. This implies that when the industry is cutting reserves rate adequacy is also generally deteriorating; when reserve strengthening is taking place then rate adequacy is improving.



We therefore have a good candidate for looking at change in rates, even where rate adequacy cannot be directly observed. A better improvement would, of course, be a standard for measuring and monitoring rate adequacy transparent to the reader of financial reports.

While the model assumptions may seem idealistic or naïve, this simple explanation of market behavior matches the empirical evidence in several ways.

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1. Given realistic assumptions on payment pattern, a cycle of fourteen to sixteen years is produced. This is close to the actual average length of a cycle in the insurance industry.
2. The model predicts that calendar year loss ratios will lag “true” accident year losses by approximately the average payment duration, and that the amplitude of the calendar year loss ratio cycle should be less than the amplitude of the “true” accident year loss ratio cycle. This is observable in industry data.
3. Given realistic assumptions on payment pattern, the one-year development test moves approximately in phase with the rate change. This is observable in industry data.
4. Industry carried IBNR for the current accident year as of twelve months is roughly a constant percent of premium. This is consistent with our model assumptions.

4. CONCLUSIONS

We have shown that sufficient conditions for creating and perpetuating a market cycle can be based on three very simple assumptions. Further, these assumptions lead to a cyclical pattern that, while idealized, is a first order approximation to what can be observed in industry financial results.

The sufficient conditions are based on the interaction of reserving based on a constant permissible loss ratio (PLR) and pricing based on assuming the reserving is exactly accurate. In other words, reserving and pricing that ignore the cycle are sufficient to produce a cycle.

If the market cycle is an undesirable phenomenon⁸, this model suggests the strategy to reduce its impact. If the cycle is caused by a failure to recognize that there is a cycle, then the solution is to better incorporate information measuring changes in rate adequacy into reserving and pricing. This means objective and auditable standards for rate monitors and public disclosure of changes in rate adequacy.

⁸ One alternative view (e.g., Feldblum 2001) is that market cycles are a sign of a rational market, and serve a healthy purpose of “maintaining long-term profits.”

Supplementary Material

An Excel spreadsheet example of the calculations in this paper is available upon request from the author.

Appendix A – Difference Equations and Market Cycle

A difference equation is a form of a recursive relationship function in which one value in a sequence is a function of prior values.

A simple first-order example of a recursive relationship is of the form below.

$$Y_j = r \cdot Y_{j-1} \tag{A.1}$$

The “solution” to this equation is to write the function in non-recursive form. For formula (A.1), the solution is given as $Y_j = r^j \cdot Y_0$, where Y_0 is some initial condition.

Difference equations are discrete versions of differential equations. They can take a linear form, such as in (A.2), or more complex non-linear forms. In our market cycle model, the calendar year losses and premiums are generated using a linear form; loss ratios include a recursive term in the denominator and are therefore generally non-linear.

$$Y_j = a_1 \cdot Y_{j-1} + a_2 \cdot Y_{j-2} + a_3 \cdot Y_{j-3} + \dots \tag{A.2}$$

The number of prior terms included in the function is the “order” of equation. A second order equation, would include Y_{j-1} and Y_{j-2} .

Much like differential equations, the general form of exact solutions for higher-order difference equation is hard to find. It is often recommended that solutions be found numerically rather than analytically; fortunately this is also the simplest approach for looking at the market cycle application.

The solutions of the linear difference equations are usually combinations of exponential and trigonometric functions. As such, they can either be in equilibrium with a constant cycle amplitude or some damped or expanding cycle.

It is useful to look at some special cases for the market cycle form where a perfect cosine function is produced as the solution. This can be done for a model in which payments are split between some amount in the first year and the remainder as a mass point in some later year m , such that $\beta_m \neq 0$, $\beta_1 = 1 - \beta_m$, and all other years’ payments are zero. This results in the following linear difference equation for $Y_j = (CYIL_j - L_j)$.

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$$Y_j = \frac{\beta_m}{n} \cdot \left((Y_{j-1} + Y_{j-2} + \dots + Y_{j-n}) - (Y_{j-m} + Y_{j-m-2} + \dots + Y_{j-m-n}) \right) \tag{A.3}$$

In order to produce an analytic solution we must set the percent paid as the mass point according to the following formula.

$$\beta_m = \frac{n \cdot \text{SIN} \left(\frac{\pi}{n+m} \right)}{2 \cdot \text{COS} \left(\frac{(n+1) \cdot \pi}{n+m} \right) \cdot \text{SIN} \left(\frac{n \cdot \pi}{n+m} \right)} \tag{A.4}$$

The resulting solution to the difference equation is below. The values *a* and *b* are arbitrary constants that can be set based on initial conditions. The result is a perfect cosine curve with period $2 \cdot (n + m)$. In other words, if we include a review of the past *n* calendar years in pricing, and the mass point of payments is in year *m* (with $m > n$), then a cycle is created that repeats every $2 \cdot (n + m)$ years.

$$Y_j = a \cdot \text{COS} \left(j \cdot \left(\frac{2 \cdot \pi}{2 \cdot (n + m)} \right) + b \right) \tag{A.5}$$

Some representative values from this formula are given in the table below. These values are not intended as estimates of insurance payment patterns; they merely give a sense of the payment durations needed to produce cycles of various periods.

Exact Stable Solutions to Difference Equations

Number of CY in Pricing n	Mass Payment at Age m	Payment Pattern									Cycle Period $2 \cdot (n+m)$	
		1	2	3	4	5	6	7	8	9		
1	2	0.00%	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	6
1	3	29.29%	0.00%	70.71%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	8
1	4	38.20%	0.00%	0.00%	61.80%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	10
2	3	10.56%	0.00%	89.44%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	10
2	4	26.79%	0.00%	0.00%	73.21%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	12
2	5	34.40%	0.00%	0.00%	0.00%	65.60%	0.00%	0.00%	0.00%	0.00%	0.00%	14
3	4	14.14%	0.00%	0.00%	85.86%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	14
3	5	25.51%	0.00%	0.00%	0.00%	74.49%	0.00%	0.00%	0.00%	0.00%	0.00%	16
3	6	32.00%	0.00%	0.00%	0.00%	0.00%	68.00%	0.00%	0.00%	0.00%	0.00%	18
4	5	15.94%	0.00%	0.00%	0.00%	84.06%	0.00%	0.00%	0.00%	0.00%	0.00%	18
4	6	24.72%	0.00%	0.00%	0.00%	0.00%	75.28%	0.00%	0.00%	0.00%	0.00%	20
4	7	30.34%	0.00%	0.00%	0.00%	0.00%	0.00%	69.66%	0.00%	0.00%	0.00%	22
5	6	17.04%	0.00%	0.00%	0.00%	0.00%	82.96%	0.00%	0.00%	0.00%	0.00%	22
5	7	24.19%	0.00%	0.00%	0.00%	0.00%	0.00%	75.81%	0.00%	0.00%	0.00%	24
5	8	29.13%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	70.87%	0.00%	0.00%	26

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Abbreviations and notations

β_i	Percent of ultimate loss paid in year (development period) i
$CYIL_j$	Calendar Year Incurred Loss in CY j
$CYPL_j$	Calendar Year Paid Loss in CY j
g	Growth rate as percent; for example 2% growth implies $(1 + g) = 1.02$
L_j	Ultimate losses in year j
n	Number of prior calendar years reviewed to set premium
PLR	Permissible Loss Ratio; assumed to be a constant amount
$Prem_j$	Charged premium in year j
$ProfitAY_j$	Ultimate Underwriting Profit in accident year j
$ProfitCY_j$	Book Underwriting Profit in calendar year j

Biography of the Author

Dave Clark is a senior actuary with Munich Reinsurance America, and part of the Actuarial Research and Modeling team.