CHAPTER ELEVEN

THE COST OF CAPITAL AN AXIOMATIC APPROACH

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OVERVIEW

The Ratemaking Principles of the Casualty Actuarial Society include the following:

> Principle 2: A rate provides for all costs associated with the transfer of risk....
>
> The rate should include a charge for the risk of random variation from the expected costs. This risk charge should be reflected in the determination of the appropriate total return consistent with the cost of capital and, therefore, influences the underwriting profit provision.

This chapter describes a way to estimate the cost of risk using data from the capital markets. We approach the problem from a business perspective as well as an economic perspective. We conclude that both perspectives lead to the same conclusion, that the cost of risk is a property of the exposure being insured, and hence can be valued directly as a percentage of the premium per unit of exposure. Once estimated in this way, the result may be expressed in terms of an imputed allocation of capital and an imputed rate of return on capital, but the cost of capital must be found from the exposure first.

Calculating the risk charge requires a description of the scenarios that lead to unexpected levels of profit or loss as well as a description of the random components of individual claims. The risk charge is a function of the broad capital markets. The same approach to calculating risk charges can be applied to financial undertakings of all kinds. When covariance with the market is minimal, the market behaves as if there were a single parameter for the risk charge. When covariance with the market is significant, it can be accommodated by adding a second parameter to the description of the capital markets.

This approach is consistent with regulation, with business practice, and with general models of economic behavior. The calculations can be put in simple spreadsheets prepared by statistical agencies and individual companies. The regulation of premium rates is simplified dramatically because the cost of capital is simply a charge per unit of exposure or a fraction of expected losses, like loss adjustment expense. The cost of real reinsurance is clearly chargeable in ratemaking because it reduces the cost of capital so much that the indicated premium rate is lower when real reinsurance is present.

A BUSINESS PERSPECTIVE

Capital has many uses. In the typical insurance transaction, most of the value of capital comes from its role in supporting the assumption of risk. As Kreps points out in Chapter 6, surplus creates capacity to bear risk, and insuring risk uses up capacity. Other uses of capital, such as rewarding entrepreneurial innovation, have a small role, if any, in insurance pricing, and insurance regulators generally have no reason to include the value of such a role in the profit provision in regulated rates.

Clearly the prices of securities in the capital markets reflect a cost of capital. If a corporation with an A rating for its debt wishes to issues bonds, it will pay a premium compared to a corporation with an AAA rating. That premium is reflected in the lower selling price for its bonds, all else equal. Bond prices vary from industry to industry, and from company to company, to reflect the risk that the coupons and principle may not be paid.

This view of the cost of capital is similar to that voiced by Justice Douglas in the Hope Natural Gas case: "...the return to the equity owner should be commensurate with returns on investments in other enterprises having corresponding risks." (Mintel, Chapter 1). As Mintel points out, "...the language used by the Supreme Court seems to require an analysis that evaluates the riskiness of the business, an ability to compare returns among different industries and a method for determining a return...."

From 1921 to about 1970, profit and contingencies in property and casualty insurance rates were generally provided for by a provision for underwriting profit of a few percent; investment income was also allowed to accrue to the insurer. Michelbacher and Roos wrote in 1970 that a "provision must be included in the rates for profit, contingencies, and catastrophes."[1] In fire insurance, the National Association of Insurance Commissioners (NAIC) recommended an underwriting profit provision of 3% from 1921 to 1949.[2] This rule worked reasonably well because short-tailed lines of insurance had a lower cost of capital and the difference between the present value of the losses and the undiscounted value of the losses was a reasonable reward for the extra risk present in long-tailed lines of business.

In the 1970's the investment yields of property-casualty insurance companies were significantly greater than the yields that had prevailed in the previous fifty years. Although the rough approximations of the NAIC's profit provision had been criticized for many years, they were simply untenable during periods of high investment yields and unexceptional risks. While in retrospect it is difficult to see why the methods used successfully in the life insurance sector were not widely adopted, the fact is they were not. Several approaches were tried, but the major changes came about because of the

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[1] MICHELBACHER AND ROOS (1970), p. 23.

[2] MICHELBACHER AND ROOS (1970), loc. cit.

automobile rate hearings in Massachusetts beginning in May, 1975.[3] During these hearings, investment income was specifically included in the ratemaking formula for regulated property-casualty insurance for the first time. The methods used were those of "Modern Financial Theory" and, in particular, the Capital Asset Pricing Model.[4]

Unfortunately, application of the Capital Asset Pricing Model generally degenerated into litigation about "the total financial need of the insurer" and how "the investment income expected to be earned" was to be measured.[5] These questions ran aground on the rocky shores of practical issues, such as:

> If no time elapsed between the date of issue and the date of loss, would there be a cost of capital? If fire insurance and liability insurance have the same risk profile, why would the time delay for the payment of liability claims affect the cost of capital?
>
> If claim payments are due, say, five years from the date of policy issue, does an increase in uncertainty about payments make the premium rate go up or down? If uncertainty makes premiums go up, does this mean there can be "negative discount rates" for insurance?
>
> If an insurance policy's results are uncorrelated with the performance of broad stock market indices, does the policy have no cost of capital? If an insurance policy's results are negatively correlated with the performance of broad stock market indices, does the policy have a negative cost of capital?

The issues underlying these questions are not implied by the Hope Natural Gas case. That case leaves in place the idea that the cost of capital is a function of the risks that are underwritten. It does not introduce the idea that the capital structure of the insurance company affects the cost of capital. It does not introduce the idea that retrospective measures of investment income play a role in ratemaking.

These questions make it clear that there is a fundamental difference between the time value of money (e.g., the present value of $1 deferred but certain) and the cost of risk. It would be far more practical, if it were correct to do so, to have the rate-maker calculate the cost of risk per unit of exposure and the time value of money at the time he or she calculates the expected losses per unit of exposure and their payment pattern. Even the use of judgment to estimate the cost of risk in a rate filing would be preferable to the use of a methodology that introduces inappropriate issues and historical information.

This business view is supported by economic theory.

[3] CUMMINS AND HARRINGTON (1987), p xiii and 120.

[4] Op. cit., p. 1

[5] MINTEL (1983), p. 186

AN ECONOMIC PERSPECTIVE

From an economic perspective, a cost is estimated to be the price that creates an equilibrium between supply and demand. The cost of capital in an insurance transaction is the equilibrium price in the capital markets for the use of capital to bear risk. Fortunately, this equilibrium price can be estimated using data from the capital markets.

One would have expected the question to be addressed from the 1950's on by the methods that economists successfully used to model the prices of other goods and services after ARROW and DEBREU (1954). Unfortunately, that approach had stalled in the 1960's. Here is Borch's summary of the development as of 1962:

> 1.1 The Walras-Cassel system of equations which determines a static equilibrium in a competitive economy is certainly one of the most beautiful constructions in mathematical economics. The mathematical rigour which was lacking when the system was first presented has since been provided by Wald (1936) and Arrow and Debreu (1954). For more than a generation one of the favourite occupations of economists has been to generalize the system to dynamic economies. The mere volume of the literature dealing with this subject gives ample evidence of its popularity.
>
> 1.2 The present paper investigates the possibilities of generalizing the Walras-Cassel model in another direction. The model as presented by its authors assumes complete certainty, in the sense that all consumers and producers know exactly what will be the outcome of their actions. It will obviously be of interest to extend the model to markets where decisions are made under uncertainty as to what the outcome will be. This problem seems to have been studied systematically only by Allais (1953) and Arrow (1953) and to some extent by Debreu (1959) who includes uncertainty in the last chapter of his recent book. It is surprising that a problem of such obvious and fundamental importance to economic theory has not received more attention. Allais ascribes this neglect of the subject to *son extrême difficulté*....
>
> 3.7 The problem on the *supply* side of a reinsurance market thus appears to be similar to the problems of maximization under restraints which occur in some production models. It is clear the problem will have a solution, at least under certain conditions.
>
> 3.8 The problems on the *demand* side are more complicated.[6]

The work of Wald, Arrow and Debreu, then, building on the Walras-Cassel equations, shows how to build a model of the equilibrium price of anything traded in a competitive

[6] BORCH (1962), p. 424, 431.

market. One begins with a model of the decisions facing the buyer and seller. One then determines the equations describing Pareto-optimal behavior. These are the supply and demand schedules that underlie equilibrium prices.

Parallel to the work of Arrow, Debreu, and Borch, practical economists in the United States had begun to employ the Efficient Market Principle as a sort of Ockham's razor to compare theory with data. The Efficient Market Principle states that the prevailing prices reflect *all* of the information available to the players in the market. Here is Stephen A. Ross addressing the Society of Actuaries in April, 1994:

> I actually trace the roots of the modern subject [of finance] back a bit further. I traced it to a wonderful, somewhat neglected article in 1937 by Cowles, who examined what we now call the efficiency of markets.... Efficient market theory lay dormant after Cowles until around the 1950s, and then it picked up steam in the 1960s and 1970s. It is the empirical basis for what we think of as modern finance. If you look closely, lurking in the background of option-pricing theory, asset-pricing models, and all of the paraphernalia of modern finance, are the fundamental intuitions of efficient market theory.... [T]he thought was that the current price was really some sense of the reflection of the consensus of all of the participants in the market. As such, it incorporates all of the information that people have.[7]

The Efficient Market Principle has been found to explain movements in prices in many markets. It is reasonable to expect any regulatory or management standards for ratemaking to reflect the Efficient Market Principle.

We begin by identifying the price behavior of insurance companies and other risk-taking firms. This is established in terms of equations of price if every firm is pursuing its Pareto-optimal price strategy. Specifically, every insurance company seeks to maximize its economic value. First we set out assumptions that are intuitive enough to be considered axioms.

THE AXIOMS

The axioms that underlie this approach to calculating the cost of capital are set out in Table 1. This list has been abridged slightly from the list in VAN SLYKE (1995).

The first three axioms are restatements of the axioms underlying Borch's theorem regarding risk transfers (BORCH (1962) and GERBER (1979)). Axiom 4 introduces the time value of money in the absence of risk. Asymptotically, as the amount of risk in a multi-period transaction diminishes toward zero, the cost of capital arising from the transfer of risk approaches that of a series of payments certain in the currency in which the transaction is denominated. Axiom 5 is the Efficient Market Principle.

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[7] ROSS (1994), p. 1-2.

TABLE 1

THE AXIOMS

1. The players are averse to risk.
2. No player would pay \$X or more than \$X to be rid of a chance of losing \$X.
3. The price of capital for the use of underwriting risk is not unduly sensitive to small changes in the descriptions of the risks that are being transferred. By "description" we mean the forecasts of cash flows under a range of scenarios, their probabilities, and their timing.
4. If there is no risk but the outcomes result in flows of currency at future times, the time value of money can be determined from the current prices of risk-free bonds.
5. In the aggregate, prices reflect all of the information available to the players.
6. No individual buyer or seller controls the cost of capital.

Axioms **4** and **5** have the effect of creating a distinction between the time value of money and the cost of risk. The time value of money has to do with the ways governments print money, finance one another's economies, and the like. The time value of money is recognized explicitly by replacing all outcomes that may be realized at future times with their equivalent values in current dollars.

COMPUTATIONS OF THE COST OF RISK

Just as the prevailing price level of real estate determines the cost of real estate to the insurance company, the prevailing price level of capital in risk-taking transactions determines the cost of capital for ratemaking purposes.

These axioms imply that the cost of risk can be found from the simultaneous solution of the following three equations:

$$\pi = E[x] - P[x]$$

$$E[x] = \sum_k \sum_j \sum_i p(k)\, p\left(x = x_i \middle| t = t_j, k\right) x_i v_{t_j}$$

$$P[x] = -\frac{\pi}{s} \ln \sum_k p(k) \exp\left[-\frac{s}{\pi} \sum_j -\frac{\pi}{s} \ln \sum_i p\left(x = x_i \middle| t = t_j, k\right) e^{-s\frac{x_i v_{t_j}}{\pi}}\right]$$

These can be read, "The risk premium, or cost of risk, is the difference between the expected present value of the cash flows and the market price of the cash flows in light of the market's aversion to risk. The expected present value is the sum, over all possible amounts of cash flow, over all periods of time, over all possible scenarios, of the cash flows, weighted by their probabilities and present value factors reflecting the value of a dollar certain to be paid at time t_j. The market price of the cash flows is the weighted average of the present values of the cash flows, with each cash amount adjusted by scaling it by the risk premium and the risk-free present value factor, exponentiating it to give additional weight to adverse outcomes; when the weighted average has been computed, the exponentiation and dollar scaling are undone to get the result in dollars. The dollars over time within a scenario are offset against one another. The resulting present values for the scenarios are adjusted for the uncertainty among the scenarios."[8]

In the absence of risk, π is zero. If a set of transactions were listed from riskiest to least risky, the value of π would decrease as one moved down the list.

SCENARIOS

The concept of scenarios is crucial. In the equations above, the scenarios are denoted by the subscript k. Each scenario is defined by a set of assumptions about the ways outcomes are linked over time. For example, there might be a high litigation scenario, a high medical inflation scenario, or a high storm frequency scenario. Within each scenario, the probabilities of outcomes are independent of the outcomes at previous times. (Technically speaking, the outcomes are conditionally independent, conditioned on the occurrence of the particular scenario.) Within each scenario, income items in one time period offset outgo items in other time periods. Within an adverse scenario, the total effect of a series of costly years is added together to reflect the large amount of capital required to support the possibility of such a scenario. These sums across time periods within specific scenarios are done in dollars, not exponentiated units.

Often there is either random fluctuation or parameter uncertainty within a given scenario. The innermost sum accounts for variation given the assumptions of the particular scenario. In practice, it is sometimes more practical to explore a large number of deterministic scenarios and put all of the uncertainty in the between-scenario risk. When this is done, the equations simplify, and there is no sum over i. In other cases, such as for fire insurance, the risk might reasonably be represented by a single scenario, a single time, and a probability distribution of outcomes. In this case, the sums over j and k fall away, and the only sum is over i.

POOLING OF RISKS, LIMITATIONS ON LEVERAGE, AND REGULATION

These equations are "scaleable"; that is, if there are twice as many units of exposure, the three values $E[x]$, $P[x]$, and π all double in value. For example, if two reinsurance companies reinsure 10 million car-years and 20 million car-years, respectively, of a quota share contract, the premium received for the larger share will be twice the premium of the

[8] John Cozzolino has named the quantity $P[x]$ the "Risk-Adjusted Value" of the cash flows. The term "economic value" has become more popular, at least in the context of management information systems. Stern Stewart & Co. has registered the service mark "EVA" to refer to its consultancy in "*Economic Value Added*". The Coca-Cola company featured a lengthy description of EVA in its 1995 annual report. See also "The Real Key to Creating Wealth" by Shawn Tully, Fortune, September 20, 1993.

smaller share. The equations are scaleable because they reflect the cost if the company must go to the capital markets to get the capital to underwrite the risk.

To the company writing many identical risks, however, the risks of individual policies are not scaleable. There is a decided advantage to risk pooling and a decided cost to excessive leverage. Although the cost of capital—in the capital markets—for underwriting \$100 million of automobile insurance premium in a year depends on the exposures and not on the capital structure of the company, for the company the marginal cost of capital might be more or less than the market average. Specifically, to the extent that the results on the \$100 million of automobile insurance are independent of the company's other financial results, the company will enjoy the benefit of risk pooling. Its marginal cost of capital will be less than indicated by the equations. On the other hand, to the extent that the results on the \$100 million of automobile insurance are positively correlated with the company's other financial results, the company will have a higher cost of capital than indicated by the equations. Its marginal cost of capital will suffer from the high leverage.

As a result, rate regulation based on a cost-of-capital formula that does not depend on which company retains the risk would lead companies to manage their leverage. This is a desirable outcome. On the other hand, requiring the cost of capital to vary to offset the effects of pooling and leverage would lead companies to employ excessive leverage or inadequate risk pooling compared to the equilibrium free-market situation. The result would be unnecessarily high premiums.

NUMERICAL EXAMPLES

It is easy to see the equations in terms of a spreadsheet to calculate the cost of risk. One simply puts in a list of possible net (after-tax, present-value) cash flows, associates a probability with each, estimates a value of s from market data, sets up the three equations, and runs the solver routine to find π. The hard work is to estimate the probabilities of the possible cash flows, but that must be done in any calculation of the cost of capital.

Table 2 shows the computation of the cost of a single unit of risk, which is defined, for these examples, to be the risk in one chance in 100 of losing \$1,000. This table includes a premium of \$100, which does not affect the value of the cost of risk but makes the illustration clearer. The cost of risk of \$116 per unit is the amount that satisfies the three equations above when the capital markets show a value of the parameter s of 0.50.

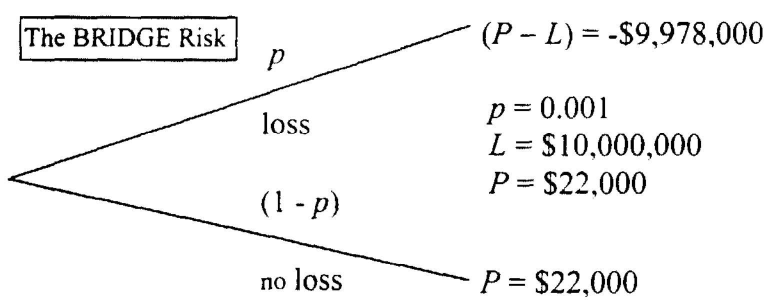
TABLE 2

CALCULATION OF THE COST OF ONE UNIT OF RISK

Market value of *s*	0.50		
Premium	126		
Loss	-1,000		
Event	Amount	Probability	Expected Value
Premium, then loss	-874	0.010	-9
Premium, no loss	126	0.990	125
			116
Economic Value			0
Expected Value			116
Cost of Risk			116

STONE (1973) examined several hypothetical insurance contracts. These were simple binomial risks that Stone developed to illustrate the basic principles.[9] Stone described a situation in which there were 2,000 identical bridges with parameters p, L, and P, each subject only to total loss, like this:

FIGURE 1



This premium of $22,000, while illustrative, is a realistic figure. If the capital markets were asked to absorb a single bridge risk like this, without any pooling, the price in the capital markets would be something like $750,000. This can be illustrated by applying the three equations to this problem. This is illustrated in Table 3.

[9] I am indebted to John Cozzolino for pointing out Stone's important work.

TABLE 3

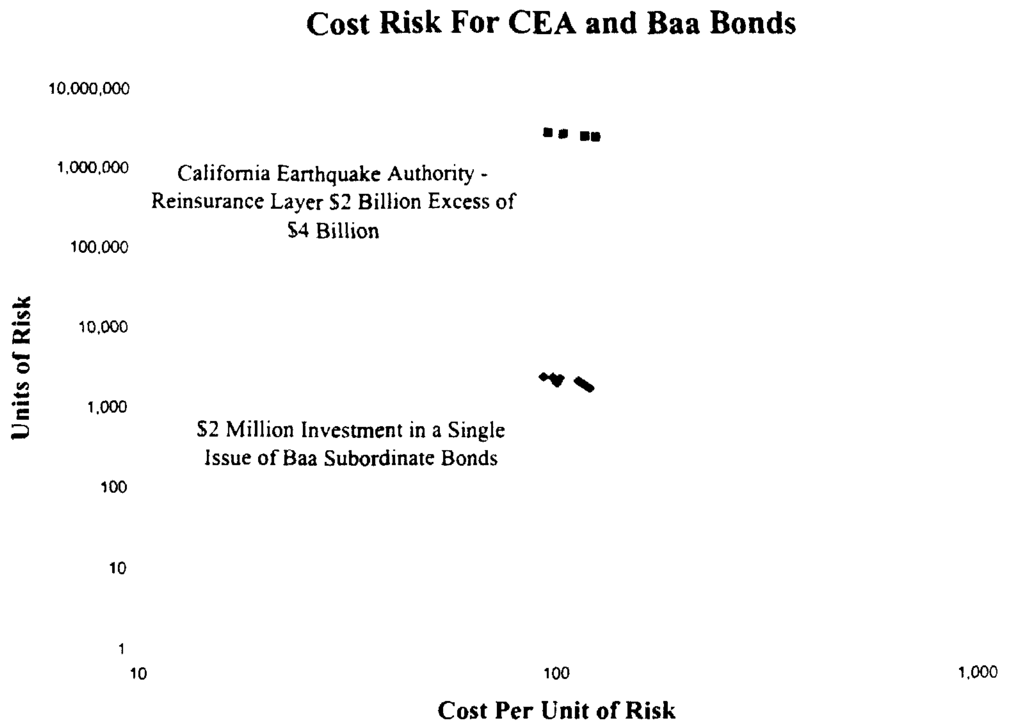
COST OF RISK FOR STONE'S BRIDGE EXAMPLE

Market value of s	0.50		
Premium	780,067		
Loss	-10,000,000		
Event	Amount	Probability	Expected Value
Premium, then loss	-9,219,933	0.001	-9,220
Premium, no loss	780,067	0.999	779,287
			770,067
Economic Value			0
Expected Value			770,067
Cost of Risk			770,067

Here the economic value is zero because at the premium of \$780,067 the company is indifferent about underwriting the bridge contract. On the other hand, if a single insurer were to take a portfolio of 2,000 such risks to the market, the premium per bridge would be only about \$13,000, less than that cited in Stone's example. The importance of pooling is discussed at length below.

RUSSO AND VAN SLYKE (1996) applied these equations to two dramatically different transactions in the capital markets, the purchase of \$2 million of Baa bonds in the bond market, and the reinsurance layer of the California Earthquake Authority (CEA), which attaches at \$4 billion plus accumulated earnings and has a policy limit of \$2 billion. Again, the unit of risk is the risk in one chance in 100 of losing \$1,000. The results are shown in Figure 2. Russo and Van Slyke reported values of s from about 0.35 to about 0.55 depending on the assumptions they used; this variation is reflected in Figure 2. While much empirical work remains to be done to get a full understanding of the market parameter s, it seems clear that its value is such that the cost of risk in one chance in 100 of losing \$1,000 is about \$100, not \$30 or \$40.

FIGURE 2



It is striking that two transactions in such widely different parts of the world-wide capital markets displayed such similar values for the cost per unit of risk. The data could have indicated two different costs per unit of risk, say $50 and $150. The data did not. This consistency in the cost per unit of risk is a result of the data, not an illusion created by the model.

The Efficient Market Principle suggests that this will be the case. Investors can invest in Baa bonds or in the stock of reinsurance companies, as well as many other investments, and they adjust their investments to reflect their expectations of risk. If there were a greater price for underwriting risk in one part of the capital market, capital would flow into that part, driving up the supply, and driving down the price.

It is precisely these estimates of the cost of risk that should be used in ratemaking.

RATEMAKING

For ratemaking purposes, these equations can be applied to the losses, while premiums and expenses can be considered separately.

The expression for the expected present value of losses can be restated in terms of the "duration" of losses and the risk-free interest rate.

$$E[x] = E[L]\{1 - D_m r_f\}$$

That is, the expected present value of the losses is the undiscounted expected value of the losses, diminished by the adjusted MacCauley duration times the risk-free rate of interest. This is an approximation, but it is adequate. There are second-order expressions for duration, but their accuracy is not necessary for ratemaking.

The net cost to the insurer to underwrite N policies with M total units of exposure at premium rate P and total premium PM is the sum of the following costs:

$ME[L]\{1 - D_m r_f\}$ where $E[L]$ is the expected loss rate per unit of exposure, without discount for risk or the time value of money

$M\pi$ where π is the average cost of risk per policy

$M U_M$ where U_M denotes those underwriting expenses that increase with the number of units of exposure

$N U_N$ where U_N denotes those underwriting expenses that increase with the number of policies

$P U_P$ where U_P denotes those underwriting expenses that increase with the policy premium P

$P C$ where C is the commission rate

$P T$ where T is the premium tax rate

The premium equation becomes:

$$P = \frac{E[L]\{1 - D_m r_f\} + \pi + U_M + \frac{N}{M} U_N}{1 - (U_P + C + T)}$$

This can be read, "The premium rate per unit of exposure is the total fixed cost per unit of exposure divided by the complement of the costs that vary with premium. The total fixed cost per unit of exposure is the expected present value of the losses at the risk-free rate of return, plus the cost of risk, plus the underwriting expenses per unit of exposure."

Today the rating bureaus and larger companies publish the values of expected losses. Determining these values requires the analysts to forecast the loss payments over time. Also, the forecasts are uncertain, and the uncertainty can be estimated from the assumptions and data used to forecast the losses. From this data the rate-maker can compute the MacCauley duration and the average cost of capital per unit of exposure. Either the rate-maker or the regulator can publish the risk-free interest rate to be used for each duration; it can easily be read from the trading prices of U. S. Treasury securities on any day.

Forecasts of undiscounted losses and arcane discussions about the appropriateness of profit should be replaced with explicit calculations of the present value of losses and the cost of capital. All of this seems remarkably out-of-step with the "Modern Theory of Finance" only because of the customs that have arisen around discussions of investment portfolios, which make it difficult to discuss the cost of capital in the way we do here. These customs are not based on an economic analysis of equilibrium prices in capital markets.

Application of the equations to determine the cost of risk, π, requires a careful description of the risks of underwriting the insurance. It does **not** require that one step in the calculation is an allocation of surplus or assets to the particular block of insurance. Of course, once one has calculated the cost of risk π to be a certain number of dollars per unit of exposure, that result can be *expressed* as a certain return on a certain amount of imputed surplus. This is merely a way of describing the cost, not a way of estimating it.

In this formulation, the capital structure of the insurance company is not relevant in determining the premium rate. There is no need to allocate capital or surplus among lines and sub-lines. Sound companies command the same premium whether they are using their capital fully or not. They have the same cost of risk when they look to the capital markets to support their underwriting.

REINSURANCE

The cost of capital is ultimately determined by the cost of the worst risks, unless they are remote. Care must be taken to identify the worst scenarios and establish realistic probabilities for those scenarios. The estimates of probabilities should be based on historical data to the extent possible. Ratemaking, determining policy terms, and risk management all go hand in hand.

Reinsurance increases the expected cost of losses and expenses by introducing the transaction costs and profit margins of the reinsurer. Reinsurance that transfers significant risk lowers the expected total of losses, expenses, and the cost of capital, however. This beneficial effect of reinsurance should be reflected in rate filings. Especially when reinsurance is effectively essential to the prudent underwriting of risks, as is typically the case for homeowners insurance, the costs of reinsurance should be reflected in the premium rates, along with the lower value of the cost of capital that is the result of the reinsurance.

Table 4 shows the cost of risk and total premium for a 50% share of two of Stone's bridge policies.

TABLE 4

COST OF RISK FOR 50% SHARE OF TWO BRIDGE POLICIES

Market value of *s*	0.50		
Premium	450,303		
Size of Each Loss	-5,000,000		
Event	Amount	Probability	Expected Value
Premium, then one loss	-4,549,697	0.0019980	-9,090
Premium, then two losses	-9,549,697	0.0000010	-10
Premium, no loss	450,303	0.9980010	449,403
			440,303
Economic Value			0
Expected Value			440,303
Cost of Risk			440,303

In this example, the cost of risk has been reduced from \$770,067 to \$440,303 even though the expected value of loss payments is unchanged. This reduction in the cost of risk has reduced the total premium from \$780,067 to \$450,303. By extending this to a Poisson process for 2,000 identical bridge contracts, the estimated premium per bridge is just \$13,951, and the total risk premium for 2,000 bridges is \$7.9 million.

Indicated premium rates will be lowest if sound reinsurance is recognized. When reinsurance does not transfer real risk, as in some window-dressing contracts, the regulator might disallow its costs. The company might reasonably be asked to justify any provisions for reinsurance that do not minimize the economic cost to the primary insurer. Except in these unusual situations, the cost of reinsurance is a legitimate insurance company expense.

EXPOSURE IN RATEMAKING

Although characterized in rate hearings as a debate between a Return on Equity school and a Return on Premium school, the real debate is about whether the measure of exposure alone can accurately capture the information about the cost of capital. Ratemaking based on exposure may reflect the particular losses and exposures of the insurer selling the insurance, but otherwise it does not depend on the firm selling the insurance.

An appropriate measure of exposure is, of course, proportional to the expected level of loss costs. As noted above, the expected present value of losses, $E[x]$, the economic value of the losses, $P[x]$, and the risk premium π are all scaleable. That is, they all change in tandem with across-the-board changes in the cash flows denoted by x. Therefore, the cost of risk is a fixed proportion of the losses.

The proportion depends on the uncertainty of the cash flows and their distribution over time. The proportion therefore varies from one kind of insurance to the next. On the other hand, the cost of risk as a proportion of expected losses does not vary among insurance companies except to the limited extent the individual insurer's operations change the probabilities of loss payments or their timing.

(The proportion also depends on the capital market's valuations of risk-free securities and the capital market's aversion to risk, both of which can be determined without reference to the ratemaking problem at hand and introduced into the ratemaking procedure as external constants at the time the rates are promulgated.)

The cost of risk is therefore a fixed amount per unit of exposure. The fixed amount depends on the forecasts of loss payments, including their timing and estimates of the possible payments and their probabilities. The cost of risk is a function only of ratemaking data, the risk-free rate of return, and the capital market's cost per unit of risk as embodied in the parameter s which determines the market's price per unit of risk.

Finally, the reduction in the cost of risk per unit exposure brought about by the pooling of many units of exposure should be reflected in the computation by applying the formulas to the volume of exposures being underwritten. For personal lines ratemaking, most of the risk in the policies subject to a given rate filing arises from parameter uncertainty or the risk of conflagration or windstorm. For personal lines, therefore, the average cost of risk per unit of risk will not vary significantly whether the loss forecasts encompass $100 million of losses or $1 billion. The cost of risk per unit of exposure is the cost of risk for the insured exposures of a representative firm divided by the number of units of exposure.

COVARIANCE WITH THE MARKET

ROSS (1976) had the great insight that risks whose outcomes are independent of the outcomes of the broad capital markets should command a lower cost of capital than risks that have outcomes that are positively correlated with the movements of broad market averages. For example, an investment that performs well when the economy is strong and performs poorly when the economy is weak is less valuable than a risk whose expected outcomes move in the opposite way.

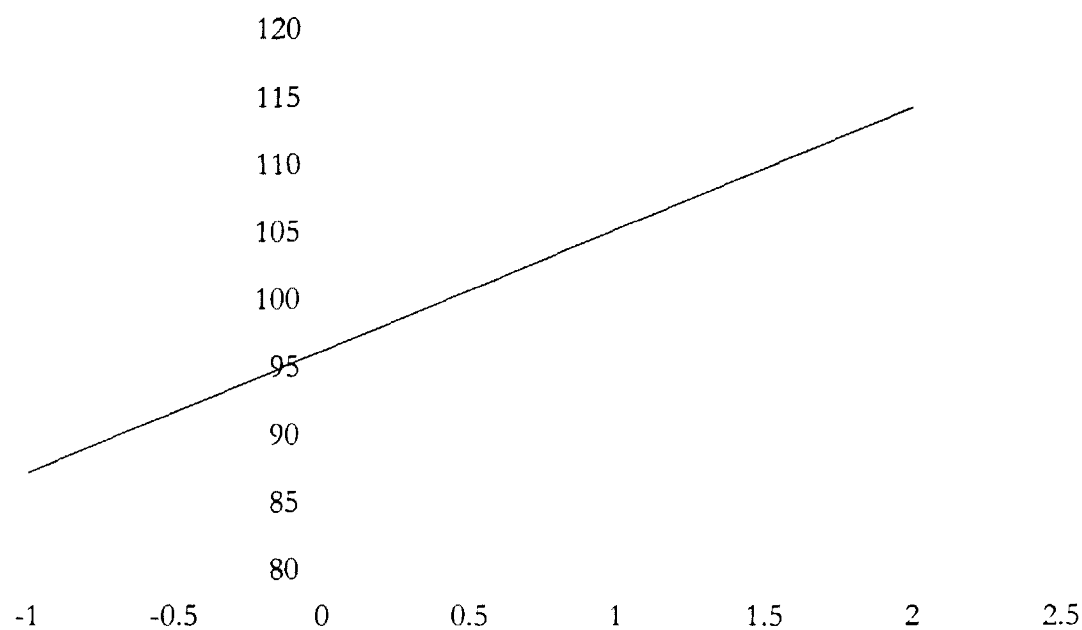
As Ross shows, the cost of capital can be expected to vary with the relationship of the risk's outcomes to a number of factors in the broad portfolio of risks in the capital market. Ross suggested the use of factor analysis to find the relationship of the cost of capital for a particular risk to a composite market factor. Factor analysis is a statistical procedure that identifies a set of constants to apply to a number of independent variables to create a composite variable that best explains the performance of an independent variable. Adopting this idea, a composite variable could be found that has the property that the market's cost per unit of risk is a linear function of that composite variable. The results would be like those shown in Figure 3.

FIGURE 3

THE CAPITAL MARKET LINE

THE COST OF RISK PER UNIT OF RISK AS A FUNCTION OF MARKET

(Illustrative example)



Composite Variable Measuring Relationship to Market

Thus, the Beta of the Capital Asset Pricing Model is replaced by a composite economic indicator particular to the risk. Risks whose outcomes move in tandem with broad market averages command a higher cost per unit of risk than risks whose outcomes are independent of broad market averages.

Note that this is contrary to the key conclusion of the Capital Asset Pricing Model that risks that are *independent* of the market have *no* cost of risk. That conclusion is a result of certain assumptions of the Capital Asset Pricing Model that do not apply in general.

Most property-casualty risks are not significantly correlated with the outcomes of the capital markets as a whole. The value of π per unit of risk can be assumed to be a constant for rate regulation purposes except for such lines as surety, which usually are not regulated.

COMPARISON TO PRACTICE

Most goods and services in most industries are priced using mark-ups that are a percentage of sales. Return-on-sales rules may not be unique to the insurance world, but return-on-investment rules are unique to the investment world.

In most jurisdictions, in most lines of property-casualty insurance, the price of insurance is set by the competitive market, either with or without the prior approval of filed rates. In these typical situations, insurers file rates that reflect a provision for profit and contingencies that is a percentage of premium. The percentage of premium is justified using data about the percentage that is charged in other states and for comparable risks in other industries. The notable exceptions are personal lines (automobile and homeowners, which are widely regulated) and the state of California (which regulates many commercial insurance products using a return-on-nominal-equity approach).

In the typical case, the provision for profit and contingencies is not computed using a return-on-equity approach. This does **not** mean that the profit provision is set without reference to the capital markets. Bingham (Chapter 4) says no body of comparative reference data exists for return-on-premium as a function of risk. In fact, insurers have available to them the return-on-premium data for hundreds of insurance companies in dozens of states in which insurance profit provisions are not regulated. This return-on-premium data is the appropriate basis for determining profit provisions when it is inappropriate to develop a full analysis of payment patterns and their probabilities.

Even in California, the Department of Insurance has adopted rules for the calculation of the rate of return that substitute nominal surplus for the company's actual surplus. The effect is that provisions for profits are set as percentages of premiums, much as Mintel (Chapter 9) describes.

Anecdotal information this writer has accumulated over the years suggests that actuaries have used measures of standard deviation and measures of the probability of ruin when using statistical methods to price risk. In either approach, the actuary tries to select a parameter that generates an appropriate risk charge as a percentage of premium. The probability-of-ruin benchmark is often equivalent in practice to a standard deviation benchmark because the actual results must depart from expected by a certain number of standard deviations to trigger the event of ruin. If the methods described here were widely used in rate filings, the effect would be that capital costs on property-casualty insurance vary roughly in proportion to the standard deviation of results for a wide range of insurance products. The charge per standard deviation would vary from product to product.

Bingham (Chapter Four) suggests the use of "operating return" instead of return on premium. This seems to be the same as return-on-premium in practice because each line will have a single ratio of operating return to premium. Bingham seems to be conceding that the company's capital structure is not *necessarily* relevant to the decision about the provision for profit. Given that one will use a method that is independent of the capital structure of the firm, the use of operating return presents at least one small challenge that does not appear when using return-on-premium directly. Operating returns are analogous to returns on assets, but in the case of very short-tail lines of insurance with high levels of risk the operating return will be quite high and quite unlike the returns on assets generated by typical investments. For example, a return-on-premium of 30% might be appropriate for property catastrophe insurance. If the duration of liabilities were 0.2 years, this would be an operating return (as defined by Bingham) of 150%. It is difficult to see how this result can be obtained by analogy with investments, no matter how correct it is. Arguing by analogy with investment returns has not worked in practice for many companies in many lines of insurance.

CRITICISM OF IRR MODELS, INCLUDING CAPM

Given the axioms listed in Table 1, internal-rate-of-return (IRR) rules are inappropriate for insurance. Yet IRR rules have served the investment community well for more than a century. It is not surprising, then, that return-on-investment rules are good approximations to the formulas above when the decision is characterized by cash flows that are outward at first, and inward later, with the uncertainty about the inward cash flows increasing over time (at a decreasing rate). All of the internal-rate-of-return (IRR) rules discussed in investment literature are special cases of the formulas shown above. Therefore IRR rules generally work well in pricing bonds. This derivation is shown in Appendix 2.

The cost of risk is always a cost. The major problem with the ratemaking methods that impute IRR is that they get the minus sign wrong. IRR methods discount losses more as the losses' riskiness increases. This isn't just counterintuitive, it is wrong. No one really thinks that riskier loss payments should be discounted more than predictable loss payments. Even D'Arcy and Bingham imply that something must be done to make their equations practical when comparing lines of equal duration and different risk.

IRR methods ask for an explicit allocation of surplus to the risks in each rate filing. As McClenahan points out in Chapter 8, "...no matter how much the rate-of-return advocate may wish to ignore the fact, there is no such thing as North Dakota Private Passenger Automobile Surplus - unless, of course, we are dealing with a company which writes North Dakota private passenger automobile insurance exclusively." There is, however, a probability distribution of outcomes for North Dakota Private Passenger Automobile which determines the average number of units of risk per car-year, and there is a cost per unit of risk in the capital markets as shown in Figure 2.

Applying approaches based on the Capital Asset Pricing Model (CAPM), even to generalized problems of asset management, is not always appropriate. CAPM has been criticized for its lack of predictive power and its restrictive assumptions. Indeed, one assumption of the Capital Asset Pricing Model is that the decision-maker is trying to optimize the performance of an infinitely divisible portfolio of equity investments. This assumption alone should cause one to wonder why CAPM should inform us about the cost of capital for insurance. Finally, ROLL and ROSS (1994) have shown that in general it is not practical to calculate the parameters of the Capital Asset Pricing Model from data about portfolios of securities.

CAPM is inconsistent with Efficient Market Principle because it values risk in proportion to the variance of returns. The mean and variance of a set of uncertain outcomes are sufficient to determine the cost of risk under the axioms in Table 1 only if the possible outcomes are normally distributed. This is rarely the case in practical situations, even for portfolio management. The widely used Black-Scholes model, for example, assumes that the logarithms of market values are normally distributed, which implies distribution of returns much more skewed than a normal distribution. In practical problems, variance loads lose a lot of information about the risks of adverse results. This criticism applies to other approaches that rely on one statistic of the probability distribution of outcomes, including those based on probability of ruin, e.g., Pierson (Chapter 5) and Kreps (Chapter 6).

Finally, one does not need the Capital Asset Pricing Model, with its elegant use of the property of the variance of a probability distribution, to get to the common-sense idea that risks whose outcomes vary with the direction of outcomes of the broad capital markets will command a higher risk premium than those that do not. Figure 3 shows one way to estimate the cost of capital for an investment or an insurance business in light of how its outcomes correspond to some composite market index.

SUMMARY

There are many implications for those who make decisions about insurance, whether as underwriters, actuaries, marketers, investors, or regulators.

There is a cost of risk for insurance companies when they underwrite a set of insurance contracts. That cost of risk is a function of the probabilities of gains and losses, with the possible gains and losses expressed at their present value. A fundamental economic analysis shows that the cost of risk is constant per unit of exposure. The number of units of risk per unit of exposure depends on the nature of the exposure that is being rated.

The "Modern Theory of Finance" is irrelevant for rate filings. Internal rate-of-return calculations play no role because the assumptions underlying such methods are not valid for insurance. Per unit of risk, the cost of risk is the same in insurance and in investments.

In most practical ratemaking situations, equations for the computation of the cost of risk can be applied to compute the cost of risk per unit of exposure. The equations rely on the same forecasts of loss payments under a range of scenarios that underlie the estimate of the average loss cost per unit of exposure. The cost of risk is found from the solution of the following three simultaneous equations, where the parameter s is found in the capital markets:

$$\pi = E[x] - P[x]$$

$$E[x] = \sum_k \sum_j \sum_i p(k) p\left(x = x_i \middle| t = t_j, k\right) x_i v_{t_j}$$

$$P[x] = -\frac{\pi}{s} \ln \sum_k p(k) \exp\left[-\frac{s}{\pi} \sum_j -\frac{\pi}{s} \ln \sum_i p\left(x = x_i \middle| t = t_j, k\right) e^{-s\frac{x_i v_{t_j}}{\pi}}\right]$$

The cost of a unit of risk can be observed in the capital markets just as the cost of bread can be observed in the markets in which bread is exchanged. It is embodied in a parameter, s. This parameter can be estimated using the equations and data about the prices at which transactions are actually priced.

Loss payments that will be paid some time after the premium is collected should be adjusted to their present value at the time the premium is collected using the **risk-free** rate of return, not some higher rate of return. The risk-free discount factor is the factor that converts currency that will be received with certainty at some future time into its value today. It can be read from the newspaper each day (and from the Internet even on weekends). Discounting at a higher rate of return, such as a company's internal profit target (return on equity), leads to an understatement of the economic cost of the losses.

Loss forecasts are uncertain. The greater the uncertainty, the greater the cost of capital, and the higher the indicated premium rate.

When data is insufficient to do the calculations explicitly, or when the desired accuracy does not merit a large amount of study, an informed estimate considering the cost of capital for other lines of insurance will be more appropriate than an informed estimate considering the yield rates on investments. Internal-rate-of-return formulas that underlie the calculations of yield rates do not apply to insurance because they rely on the **assumption** that cash flows are outward first and inward later. This assumption applies to investments, but not to insurance.

Because the capital markets are vastly larger than any one risk, the cost of risk is directly proportional to the size of the risk. If the cost of risk in underwriting \$20 million of auto insurance is \$1 million, the cost of risk in underwriting \$40 million of auto insurance is \$2 million. This means that the **cost of risk is a percentage of the premium**

underwritten, and the percentage varies from one kind of insurance to the next depending on the riskiness of the kind of insurance. (Precisely the same statements can be said about the markets for commodities, or common stocks, or any other type of risk.)

To a specific company, the cost of risk depends on all of the company's assets and liabilities. Unless regulators interfere with the capital markets, the company's other assets and liabilities and the capital markets' cost for one unit of risk will affect the company's willingness to extend its underwriting leverage in a way that optimizes the competitiveness of the insurance markets.

After a careful analysis, there is no reason to adopt methods based on internal-rate-of-return calculations. A careful exposition of the problem, using a minimal set of assumptions in the tradition of Arrow and Debreu, leads to the same simple conclusions that McClenahan, Mintel, and Toney have suggested for other reasons.