Exam 8

CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

Expertise. Insight. Solutions.

Exam 8

Advanced Ratemaking

October 28, 2016

4 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 53.25 point examination consists of 21 problem and essay questions.

2. For the problem and essay questions, the number of points for each full question and part of a question is indicated at the beginning of the question or part. Answer these questions on the lined sheets provided in your Examination Envelope. Use dark pencil or ink. Do not use multiple colors or correction fluid/tape.

   - Write your Candidate ID number and the examination number, 8, at the top of each answer sheet. For your Candidate ID number, four boxes are provided corresponding to one box for each digit in your Candidate ID number. If your Candidate ID number is fewer than 4 digits, begin in the first box and do not include leading zeroes. Your name, or any other identifying mark, must not appear.

   - Do not answer more than one question on a single sheet of paper. Write only on the front lined side of the paper – **DO NOT WRITE ON THE BACK OF THE PAPER**. Be careful to give the number of the question you are answering on each sheet. If your response cannot be confined to one page, please use additional sheets of paper as necessary. Clearly mark the question number on each page of the response in addition to using a label such as "Page 1 of 2" on the first sheet of paper and then "Page 2 of 2" on the second sheet of paper.

   - The answer should be concise and confined to the question as posed. When a specified number of items are requested, do not offer more items than requested. For example, if you are requested to provide three items, only the first three responses will be graded.

   - In order to receive full credit or to maximize partial credit on mathematical and computational questions, you must clearly outline your approach in either verbal or mathematical form, showing calculations where necessary. Also, you must clearly specify any additional assumptions you have made to answer the question.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

4. Prior to the start of the exam you will have a **fifteen-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. A chart indicating the point value for each question is attached to the back of the examination. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.

   - Verify that you have received the reference materials:

   a. National Council on Compensation Insurance, Experience Rating Plan Manual for Workers Compensation and Employers Liability Insurance (*Excerpt from 2016 Study Kit*).
   b. Insurance Services Office, Inc., Commercial General Liability Experience and Schedule Rating Plan.
   c. National Council on Compensation Insurance, Retrospective Rating Plan Manual for Workers Compensation and Employers Liability Insurance (*Excerpt from 2016 Study Kit*).

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until two hours after the start of the examination. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.

7. At the end of the examination, place all answer sheets in the Examination Envelope. Please insert your answer sheets in your envelope in question number order. Insert a numbered page for each question, even if you have not attempted to answer that question. Nothing written in the examination booklet will be graded. Only the answer sheets will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope. Interoffice mail is not acceptable.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by November 14, 2016.

END OF INSTRUCTIONS

1. (2.75 points)

A group of insureds have different expected claim frequencies. The number of insureds claim-free for the past *t* years is as follows:

Expected Claim Frequency	t=0	t=1	t=2	t=3
0.05	50,000	47,500	45,000	44,000
0.10	50,000	45,000	43,000	36,000
0.20	25,000	20,500	16,500	14,000
Total	125,000	113,000	104,500	94,000

Determine whether the variation of an individual insured's chance for an accident changes over time.

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2. (2 points)

a. (0.5 point)

Describe the purpose of clustering as it pertains to risk classification.

b. (0.5 point)

Briefly contrast hierarchical and non-hierarchical clustering methods.

c. (1 point)

An insurer wants to increase the number of groupings for a particular class within their classification plan. According to the American Academy of Actuaries "Risk Classification Statement of Principles," describe two considerations in using a hierarchical or non-hierarchical clustering method to determine the groupings.

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3. (2 points)

A private passenger automobile insurer sells policies through two distribution channels: independent insurance agents and directly to consumers via the internet.

The company's rates already incorporate expense differentials between the two channels but the head of sales has asked their actuary to file different pure premium factors for the two groups.

The actuary wishes to evaluate the acceptability of the request against the American Academy of Actuaries "Risk Classification Statement of Principles."

a. (1 point)
Identify and briefly describe two considerations supporting inclusion of distribution channel in the pure premium factors.

b. (1 point)
Identify and briefly describe two considerations against inclusion of distribution channel in the pure premium factors.

CONTINUED ON NEXT PAGE

4. (3 points)

An actuary is conducting a generalized linear model (GLM) analysis on historical personal automobile data in order to develop a rating plan.

a. (1.5 points)

Argue against the following factors being included as predictors in the actuary's GLM analysis:

i. Limit of liability.
ii. Number of coverage changes during the current policy period.
iii. ZIP code of the garaging location of the automobile.

b. (1 point)

The actuary is modeling pure premium with a log-link function and a Tweedie error distribution ($1 < p < 2$). Provide two arguments against the inclusion of deductible as a predictor in the actuary's GLM analysis.

c. (0.5 point)

Other than including deductible as a predictor in the GLM, describe how to determine deductible relativities and how such relativities can be incorporated in a GLM.

CONTINUED ON NEXT PAGE

5. (2.25 points)

A GLM has been used to develop an insurance rating plan. The results are given below:

Risk	Model Predicted Loss	Actual Loss
1	2,000	2,050
2	500	220
3	1,500	1,480
4	800	850
5	200	400

a. (1.75 points)

Plot the Lorenz curve for this rating plan. Label each axis and the coordinates of each point on the curve.

b. (0.5 point)

Briefly describe how the Gini index is calculated and what the Gini index measures when applied to an insurance rating program. Do not calculate the Gini index.

CONTINUED ON NEXT PAGE

6. (2.5 points)

An actuary has constructed a three-variable Tweedie GLM with a log-link function to estimate loss ratios for commercial property new business. The actuary wants to create a second model for renewal business that will include all of the variables from the new business model, plus a variable for the prior year claim count. The actuary requires that the coefficients of the variables (Average Building Age, log(Manual Premium), and Location Count) are consistent between the new and renewal models. The fitted new business model parameters are as follows:

Variable	**Name**	**Estimate**
	intercept	0.910
Average Building Age (Years)	age	0.013
log(Manual Premium)	logprem	-0.187
Location Count	loccnt	0.062

a. (0.75 point)
Calculate the modeled loss ratio for a new business policy with a manual premium of \$25,000, an average building age of four years, and having eight locations.

b. (0.75 point)
Briefly describe how to produce the renewal business model, and specify the resulting equation for the renewal business modeled loss ratio.

c. (1 point)
Identify and briefly describe two techniques that the actuary can use to assess the stability of the new variable in the renewal business model.

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7. (1.5 points)

A company is considering modifying its rating plan to include factors by age group. Below are statistics for the base model and for the new model.

Statistic	Base Model	New Model
Loglikelihood	-750	-737.5
Deviance	500	475
Parameters	10	15
Data points	1,000,000	1,000,000

a. (1 point)
Calculate the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) for both models.

b. (0.25 point)
Explain whether AIC or BIC is a more reliable test statistic as an indicator of whether to adopt the new model.

c. (0.25 point)
Recommend and briefly justify whether to adopt the new model.

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8. (2.75 points)

Given the following average severity, an actuary wants to validate if anti-selection impacts its Increased Limits Factors for a given insurance coverage:

Severity limited to:	**Policy Limit = $25,000**	**Policy Limit = $50,000**
$10,000	$4,000	$6,000
$25,000	$6,500	$8,000
$50,000	$9,000	$10,500

50% of the policies have a $25,000 policy limit and 50% of the policies have a $50,000 policy limit.

a. (1.25 points)
Demonstrate if anti-selection impacts the ILFs.

b. (1.5 points)
Identify and briefly describe two possible forms of anti-selection for ILFs and give one example for each.

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9. (1.5 points)

An actuary and an underwriter are discussing a Worker's Compensation book of business.

a. (0.5 point)
The underwriter is considering not renewing a risk in the book because the experience modifier of the account has increased. The underwriter notes that the expected loss ratio (ELR) for the predominant class code on the account has decreased over the last three years. Discuss whether the underwriter should non-renew the risk given the change in ELR.

b. (0.5 point)
A different account had large losses last year and now has a debit modification. The underwriter wants to renew this account since the debit mod will help recoup losses. Evaluate the underwriter's reasoning to renew the account.

c. (0.5 point)
The underwriter points out that the book of business has an overall experience modifier below 1.0 and believes this is due to superior risk selection. Provide two reasons why this may not be the case.

CONTINUED ON NEXT PAGE

10. (2.5 points)

An insured is subject to experience rating under the National Council on Compensation Insurance (NCCI)'s Experience Rating Plan Manual for Workers Compensation and Employers Liability Insurance. The following information about the insured is given:

Payroll (Experience Period)	$5,000,000
State	Alabama
Class	7705

The following claims apply to the experience period. Each claim involves only one person, and none are disease claims:

Claim Number	Type	Loss
1	Indemnity	29,000
2	Medical	30,500
3	Indemnity	90,000
4	Indemnity	1,500
5	Medical	45,000

Calculate the experience modification for this insured.

CONTINUED ON NEXT PAGE

11. (3.75 points)

Given the following information about 10 risks for a manufacturing class:

Risk	Type of Manufacturing	Manual Premium	Losses	Current Modification	Proposed Modification
1	Uses Robots	1,000	500	0.65	0.50
2	Uses Robots	1,000	600	0.75	0.55
3	Made by hand	1,000	700	0.70	0.70
4	Uses Robots	1,000	800	1.00	0.75
5	Uses Robots	1,000	900	0.90	0.95
6	Made by hand	1,000	1,100	1.15	1.05
7	Uses Robots	1,000	1,200	1.10	1.25
8	Made by hand	1,000	1,500	1.25	1.50
9	Made by hand	1,000	1,600	1.20	1.75
10	Made by hand	1,000	1,800	1.30	2.00

a. (3 points)

Evaluate the proposed experience rating plan compared to the current plan.

b. (0.75 points)

Evaluate the classification plan for this class.

CONTINUED ON NEXT PAGE

12. (2.5 points)

An actuary is given the following sample of experience from a grouping of five similarly-sized risks:

Risk	Actual Loss	Risk	Actual Loss
1	$85,000	4	$63,750
2	$127,500	5	$106,250
3	$42,500		

a. (1.5 points)
Construct a Table M of insurance charges and savings at entry ratios of 0 to 1.50 in multiples of 0.25.

b. (0.25 points)
Briefly describe what the insurance charge at an entry ratio of 1.25 reflects.

c. (0.75 points)
Suppose the Table M constructed above is used to price a book of Worker's Compensation retrospectively rated business. The following table of actual losses reflects the experience of this book:

Risk	Actual Loss	Risk	Actual Loss
1	$12,000	4	$106,250
2	$42,500	5	$275,000
3	$63,750		

Evaluate the appropriateness of using the Table M constructed in part a. above. Provide two reasons in support of the conclusion.

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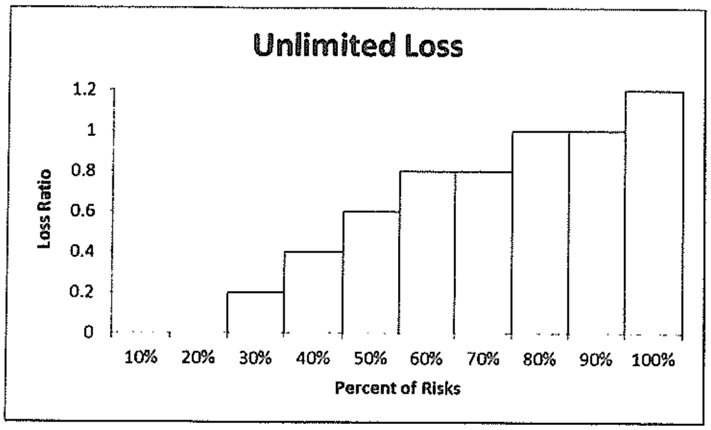
13. (2.5 points)

A risk is written using a retrospective rating plan with the following characteristics:

Standard Premium	$10,000,000
Expected Loss Ratio	60%
Loss Ratio at Maximum Premium	80%
Loss Ratio at Minimum Premium	20%
Loss Conversion Factor	1.085
Provision for Losses and Total Expenses Exclusive of Taxes	0.97

The following Lee Diagram depicts actual experience from a sample of similarly-sized risks and similar to the risk in question:



a. (1.25 points)

Determine the converted insurance charge for this plan.

b. (1.25 points)

The insured's actual ultimate losses are $8,700,000 and the final retrospective premium is $12,500,000. Determine the tax multiplier that was used in the rating of this plan.

CONTINUED ON NEXT PAGE

14. (1.75 points)

A retrospective rated policy has both a loss limitation and a maximum premium.

a. (0.5 point)
Demonstrate how the charges for the loss limitation and the maximum premium overlap.

b. (0.5 point)
Explain how the overlap is handled differently when using Table M versus Table L.

c. (0.75 point)
An actuary is pricing a large worker's compensation policy with a fixed loss limit of \$100,000. Recommend a table of insurance charges, Table M or Table L, that the actuary should use and provide two reasons supporting the recommendation.

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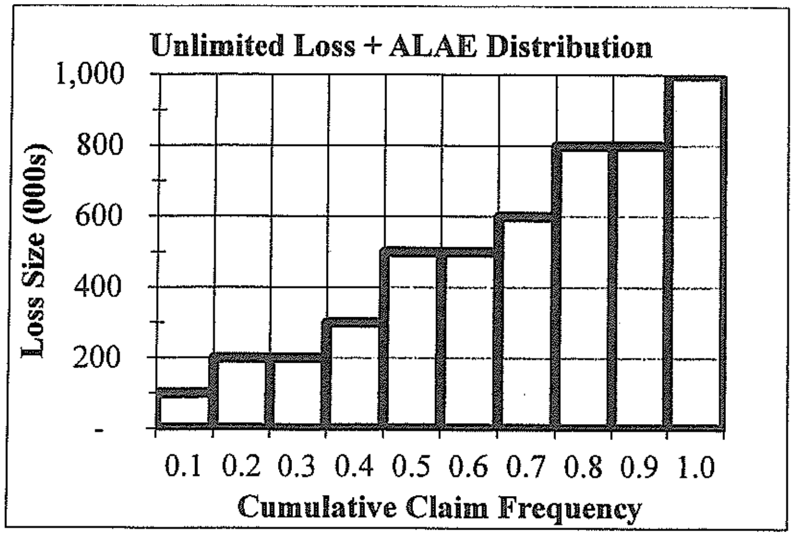
15. (1.75 points)

Given the following information about a worker's compensation book of business:



Standard premium	$840,000
Loss based assessment factor	2.0%
Ratio of ULAE to Loss	5.0%
General overhead expense	2.0%
Risk load for credit risk	4.0%
Acquisition expense	5.0%
Taxes	3.0%
Profit	-8.8%

a. (1.25 points)

Calculate the deductible for an excess workers compensation policy that minimizes the insured's loss retention with a maximum premium of $100,000.

b. (0.5 point)

Contrast the profit load in a large dollar deductible (LDD) policy to the profit load in an excess workers compensation policy.

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16. (3.75 points)

An actuary is given the following expiring policy information for a Workers' Compensation Large Dollar Deductible policy:

Expected Total Loss & ALAE	\$500,000
Deductible	\$100,000
Percentage of Loss & ALAE Excess of \$100,000	40%
Percentage of Loss & ALAE Excess of \$200,000	20%
ULAE	5%
Loss Based Assessment Factor	3%
Profit and Variable Expenses	17%
Fixed Expense	\$15,000
Aggregate Deductible	\$300,000

There are no changes to these expenses and profit provision. The table of Insurance Charges is displayed below:

Modified Table M for Similarly Sized Policies				
	Deductible			
Entry Ratio	\$100,000	\$200,000	\$300,000	\$400,000
0.5	.450	.480	.495	.505
1.0	.330	.350	.365	.370
1.5	.270	.303	.325	.350
2.0	.185	.240	.260	.286

The insured would like to retain more risk and requests the price of the following options:

Option 1: Deductible of \$200,000 with an aggregate deductible of \$400,000.

Option 2: Excess Policy with self-insured retention of \$200,000 on a per occurrence basis.

<<QUESTION 16 CONTINUED ON NEXT PAGE>>

a. (2.75 points)

Calculate the difference in price between Option 1 and the expiring structure. Use linear interpolation as needed.

b. (1 point)

The insurer observed an upward trend of ground up losses in the insured's industry for the most recent year, but is unsure if this trend will continue in the future. Based on this observation, recommend one of the two options for the insurer and fully support the recommendation.

CONTINUED ON NEXT PAGE

17. (2.5 points)

The following table contains historical adjusted Worker's Compensation loss experience:

Loss Size	Number of Claims
<$100,000	62
$100,000	23
$200,000	7
$300,000	5
$500,000	2
$1,000,000	1

Mean claim size for losses less than $100,000: $38,000.

Excess ratios were calculated directly from data for limits of $100,000 or less. For higher limits a mixed Pareto-Exponential curve was fit to losses truncated and shifted at $100,000, then normalized to mean unity. Selected values of the fitted curve of these excess ratios are shown in the table below.

Entry Ratio	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
Excess Ratio	0.92	0.83	0.76	0.69	0.63	0.58	0.53	0.49	0.45	0.41	0.38	0.35

a. (1.5 points)

Calculate an excess ratio for a limit of $250,000.

b. (1 point)

An actuary is pricing for Excess Worker's Compensation Coverage. Evaluate the impact to the excess premium under the following alternative methodologies:

- Changing the selected truncation point from $100,000 to $300,000.
- Fitting losses using a Pareto Curve.

CONTINUED ON NEXT PAGE

18. (2 points)

Catastrophe models were built to assist the insurance industry in quantifying the risk of natural disasters.

a. (1 point)
For any two of the four basic modules of a catastrophe model, provide an example of epistemic risk.

b. (1 point)
For any two of the four basic modules of a catastrophe model, provide an example of aleatory risk.

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19. (3 points)
In order for an insurance company to increase its return on capital, two reinsurance options are being considered:

I. \$5 million excess of \$5 million per risk:
- ALAE pro-rata
- Rate = 18% of premium

II. 20% Quota Share:
- Ceding commission of 30%
- Maximum ceded loss ratio of 150%

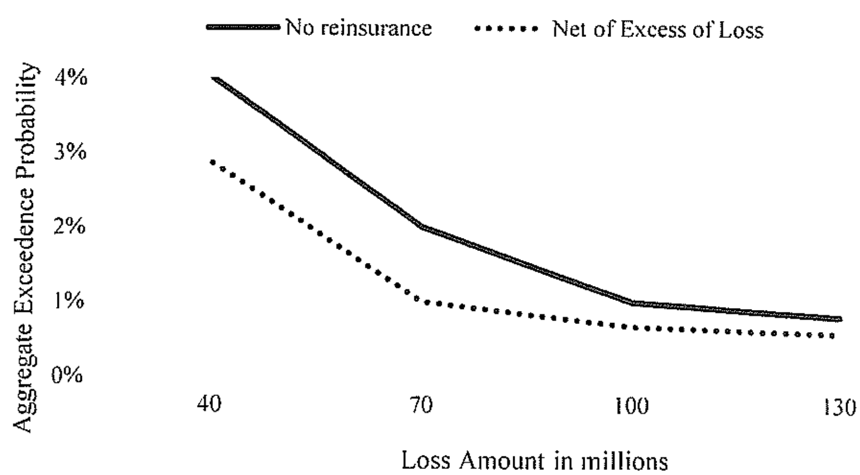
The insurance company must hold capital to support a 1-in-100 year Probable Maximum Loss. Additionally, the following information regarding the insurance company's performance last year is given:

- Premium: \$50,000,000
- Expenses: \$15,000,000
- Total Loss & ALAE: \$30,000,000
- Return on Capital: 5%

Claims greater than \$5,000,000

	Loss	ALAE
Claim 1	\$7,500,000	\$1,500,000
Claim 2	\$10,000,000	\$500,000

The Aggregate Exceedance Probability curve for the insurance company is shown below:



Determine the impact each reinsurance option would have had on last year's return on capital. Ignore investment income and taxes.

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20. (3 points)

A primary insurer has entered into property catastrophe excess of loss treaties with three reinsurers. The terms of the treaties are as follows:

Reinsurer A: 100% of $100 excess of $200 written at a nominal rate on line of 10%

Reinsurer B: 100% of $100 excess of $300 written at a nominal rate on line of 7%

Reinsurer C: 100% of $100 excess of $400 written at a nominal rate on line of 4%

Each treaty specifies a single mandatory reinstatement that is 100% pro rata as to amount. The primary insurer incurs three covered loss events during the contract period:

Loss Number	Loss Amount
1	$380
2	$260
3	$600

a. (0.75 point)

For each loss event, calculate the amount of loss retained by the primary insurer.

b. (2.25 points)

For each loss event, calculate the amount of reinstatement premium owed by the primary insurer to each of its reinsurers.

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21. (4 points)

An insurance company insures high value homes and plans to increase the maximum property value they will insure next year. The company is considering purchasing a new \$4,000,000 excess of \$4,000,000 reinsurance treaty. The reinsurer is given the following limit profile:

Insured Value Range	Experience Period On-Level Premium	Treaty Subject Premium
\$1,000,000 to \$4,000,000	\$100,000,000	\$25,000,000
\$4,000,001 to \$8,000,000	\$0	\$ 5,000,000

A reinsurance actuary has trended and developed the portfolio's historical losses for the Experience Period:

Loss Size	Number of Claims	Experience Period Ground-up Loss
Less than \$1,000,000	200	\$22,000,000
Greater than \$1,000,000	10	\$18,000,000
Total	210	\$40,000,000

The actuary is unsure if a Swiss Re Y_3 or Y_4 exposure curve is a better fit. The following MBBEFD exposure curve formulas are available along with the following information:

$$G(x) = \frac{\ln\left[\frac{(g-1)b+(1-gb)b^{x}}{1-b}\right]}{\ln(gb)}$$

$$b(c) = e^{3.1-.15c(1+c)}$$

$$g(c) = e^{c(.78+.12c)}$$

% of Insured Value	% of Cumulative Loss	
	Y_3	Y_4
25%	60%	73%
40%	72%	82%
100%	100%	100%

Given a proposed treaty rate of 1% of total subject premium, calculate the expected ceded loss ratio for the new treaty.

END OF EXAMINATION

Exam 8
Advanced Ratemaking

POINT VALUE OF QUESTIONS

QUESTION	VALUE OF QUESTON	SUB-PART OF QUESTION (a)	(b)	(c)	(d)	(e)	(f)	(g)
1	2.75	2.75						
2	2.00	0.50	0.50	1.00				
3	2.00	1.00	1.00					
4	3.00	1.50	1.00	0.50				
5	2.25	1.75	0.50					
6	2.50	0.75	0.75	1.00				
7	1.50	1.00	0.25	0.25				
8	2.75	1.25	1.50					
9	1.50	0.50	0.50	0.50				
10	2.50	2.50						
11	3.75	3.00	0.75					
12	2.50	1.50	0.25	0.75				
13	2.50	1.25	1.25					
14	1.75	0.50	0.50	0.75				
15	1.75	1.25	0.50					
16	3.75	2.75	1.00					
17	2.50	1.50	1.00					
18	2.00	1.00	1.00					
19	3.00	3.00						
20	3.00	0.75	2.25					
21	4.00	4.00						
TOTAL	53.25							

EXAM 8 FALL 2016 SAMPLE ANSWERS AND EXAMINER'S REPORT

GENERAL COMMENTS:

- Candidates should note that the instructions to the exam explicitly say to show all work; graders expect to see enough support on the candidate's answer sheet to follow the calculations performed. While the graders made every attempt to follow calculations that were not well-documented, lack of documentation may result in the deduction of points where the calculations cannot be followed or are not sufficiently supported.
- Candidates should justify all selections when prompted to do so. For example, if the candidate selects an all year average and the question prompts a justification of all selections, a brief explanation should be provided for the reasoning behind this selection. Candidates should note that a restatement of a numerical selection in words is not a justification.
- Incorrect responses in one part of a question did not preclude candidates from receiving credit for correct work on subsequent parts of the question that depended upon that response.
- Candidates should try to be cognizant of the way an exam question is worded. They must look for key words such as "briefly" or "fully" within the problem. We refer candidates to the Future Fellows article from December 2009 entitled "The Importance of Adverbs" for additional information on this topic.
- Some candidates provided lengthy responses to a "briefly describe" question, which does not provide extra credit and only takes up additional time during the exam.
- Candidates should note that the sample answers provided in the examiner's report are not an exhaustive representation of all responses given credit during grading, but rather the most common correct responses.
- Candidates should read each question carefully and answer the question as it is presented.
- In cases where a given number of items were requested (e.g., "three reasons" or "two scenarios"), the examiner's report often provides more sample answers than the requested number. The additional responses are provided for educational value, and would not have resulted in any additional credit for candidates who provided more than the requested number of responses. Candidates are reminded that, per the instructions to the exam, when a specific number of items is requested, only the items adding up to that number will be graded (i.e., if two items are requested and three are provided, only the first two are graded).

EXAM STATISTICS:

- Number of Candidates: 791
- Available Points: 53.25
- Passing Score: 37.25
- Number of Passing Candidates 301
- Raw Pass Ratio: 38.05%
- Effective Pass Ratio: 40.13%

QUESTION: 1	
TOTAL POINT VALUE: 2.75	LEARNING OBJECTIVE(S): A1
SAMPLE ANSWERS	

Sample 1

Given:

Expected Claim Frequency	t=0	t=1	t=2	t=3
0.05	50,000	47,500	45,000	44,000
0.10	50,000	45,000	43,000	36,000
0.20	25,000	20,500	16,500	14,000
Total	125,000	113,000	104,500	94,000

Calculate claims at time t:

Claims	t=0	t=1	t=2	t=3
0.05	2,500 = .05 x 50,000	2,375	2,250	2,200
0.10	5,000	4,500	4,300	3,600
0.20	5,000	4,100	3,300	2,800
Total	12,500	10,975	9,850	8,600

Calculate average frequency at time t:

0.1000 = 12,500 / 125,000	0.0971	0.0943	0.0915

Calculate frequency relative to t=0:

1.0000	0.9712 = 0.0971 / 0.1000	0.9426	0.9149

Credibility:

0.0288 = 1 - 0.9712	0.0574	0.0851

Credibility relative to t=1:

1.9963 = 0.0574 / 0.0288	2.9591

Variation of insureds' chances of accident are stable if credibility is proportional to the number of years of experience. Since the ratios of credibility are very nearly equal to 3 and 2, we conclude that the variation of an insured's chance of accident is not changing over time.

Sample 2

	(1)	(2)	(3)	(4)	(5)
n	# Claim free n or more years	Expected Claims	Frequency	Relative Frequency	Z
3	94,000	8,600	0.0915	0.9525	0.0475
2	198,500	18,450	0.0929	0.9677	0.0323
1	311,500	29,425	0.0945	0.9835	0.0165
Total	436,500	41,925	0.0960	1	

Expected claims:

- t=3: 44,000 x 0.05 + 36,000 x 0.10 + 14,000 x 0.20 = 8,600
- t=2: 45,000 x 0.05 + 43,000 x 0.10 + 16,500 x 0.20 = 9,850
- t=1: 47,500 x 0.05 + 45,000 x 0.10 + 20,500 x 0.20 = 10,975
- Total: 186,500 x 0.05 + 174,000 x 0.10 + 76,000 x 0.20 = 41,925

(3) = (2)/(1)
(4) = (3)/$(3)_{\text{Total}}$
(5) = 1 – (4)

If the variation of an insured's chance for an accident is not changing over time, then the 3-year credibility/1-year credibility will be approximately equal to 3 and the 2-year credibility/1-year credibility will be approximately equal to 2.

3+ year Z / 1+ year Z = 0.0475 / 0.0165 = 2.88
2+ year Z / 1+ year Z = 0.0323 / 0.0165 = 1.96
The ratios are approximately 3 and 2; the chance for accident is stable.

Sample 3

Credit was given for an approach that evaluated the correlation between different lags for either the relative number of insureds in each class or the frequency at each time period.

Correlation between relative number of insured in each class at different lags:

- Calculate relative distribution of insured by class (note that total insureds by class could be used for the approach below and will result in the same correlation values and conclusions):

Expected Claim Frequency	t=0	t=1	t=2	t=3
0.05	40%=50K/125K	42%	43%	47%
0.10	40%	40%	41%	38%
0.20	20%	18%	16%	15%
Total	100%	100%	100%	100%

- Calculate correlations between each lag vector, and calculate averages by lag 1, 2, and 3:

lag = 1	t=0 & t=1	t=1 & t=2	t=2 & t=3	Average
	0.9965	0.9998	0.9806	0.9923
lag = 2	t=0 & t=2	t=1 & t=3		Average
	0.9980	0.9845		0.9912
lag = 3	t=0 & t=3			Average
	0.9663			0.9663

- Determine whether the correlation is decreasing as the lag length is increasing. In the above example, this is true. Therefore we conclude that the parameter is changing over time.

Sample 4

Credit was also given to students that used the correlation approach but calculated expected claim counts, or actual frequencies, and then calculated whether these correlations were changing over time. The correlations for both are shown below. In both cases the student will also conclude that the correlation is changing as the time lag increases, and that therefore the risk parameters are changing.

Claim count calculation:

Average correlation test using calculator tables:
For lag 1 = r(0,1) = 0.9842; r(1,2)=0.9456; r(2,3)=0.9954; average=0.9750
For lag 2 = r(0,2) = 0.8730; r(1,3) = 0.9909; average = 0.8914
For lag 3 = r(0,3) = 0.8220; average = 0.8220

Downward trending average r correlation as lag increases.

Conclusion: Yes, variation of insured's chance of an accident is changing

Actual frequency calculation:

Actual Claim Free Frequency	**t=0 to t=1**	**t=1 to t=2**	**t=2 to t=3**
0.05	0.9500 =4,750/5,000	0.9474	0.9778
0.10	0.9000	0.9556	0.8372
0.20	0.8200	0.8049	0.8485

Calculate correlations between lags:
For lag 1, corr(t1, t2) = 0.903; corr(t2,t3) = 0.39; average of **0.646**
For lag 1, corr(t1, t3) = **0.748**

<u>*Sample 5*</u>
Partial credit was also given to students that stated that the Chi Squared test may be used.

Do a Chi Squared test with Chi Squared = $\sum(Actual - Expected)^2/Expected$
Across 12 cells with 11 degrees of freedom. If we reject, that means the parameters are changing over time.

EXAMINER'S REPORT

Candidates were expected to use credibility concepts to evaluate underlying risk parameters that may be changing over time. Candidates could demonstrate competency by applying a relative credibility approach as well as other approaches such as correlation between increasing time lags.

In general, candidates either applied the relative credibility approach from Bailey and Simon or applied the correlation test from Mahler's "Shifting Risk Parameters". Application of these methods to the data was relatively straightforward, and several slightly different approaches were given credit.

A common mistake was using a strict actual versus expected, or variance approach, which does not directly address whether the underlying risk parameter is shifting over time.

QUESTION: 2	
TOTAL POINT VALUE: 2	**LEARNING OBJECTIVE(S): A1**
SAMPLE ANSWERS	
Part a: 0.5 point	
• Risks are clustered into homogeneous groups in order to increase credibility when calculating expected loss costs for a risk classification. • The purpose of clustering is to classify a rating variable into different classes in a way that the variance between classes is maximized and the variance within each class is minimized. • To group classes with similar risk characteristics or expected costs into larger groups that will have sufficient volume to be used for credible ratemaking (e.g. hazard groups for NCCI excess ratios).	
Part b: 0.5 point	
• For hierarchical, to go from k to k+1 groups you must break one of the k groups into two. In other words, each set of clusters is a "nested" version of another. Nonhierarchical clustering has no constraint on how the groups are formed. • Hierarchical means that during the iterative clustering process new groups come from a single previous group (subgroups). Non-hierarchical doesn't have this limit and new groups can come from multiple previous groups. • Hierarchical clustering splits risks based on existing clusters. These splits may not be mixed or combined. For example, a group of risks is split into clusters A and B. Risks in A can be further split, but these new groups may not include any risks that were initially put in B. In this way, the method is iterative. Nonhierarchical clustering seeks to create the best k clusters, regardless of what the best k-1 clusters were. Risks from different clusters may be recombined as the groupings increase.	
Part c: 1 point	
<u>*Sample considerations (candidates were asked to provide two to receive full credit)*</u> • Credibility: Will the resulting groups be credible? As you add groups, each one becomes smaller. While this could help improve homogeneity within classes, you don't want to sacrifice too much credibility => balance these two • Expense: the cost of increasing the number of groupings should not outweigh the benefit of the new groupings • Predictive Stability: Should respond to changes in expected costs, but not change too much from year to year • Availability of Coverage: Good risks and bad risks should be equally profitable to insurers, ensuring that they will want to provide coverage • Homogeneity: Classes should have similar risk characteristics and have similar expected costs. We want to subdivide groups in a way that makes them more similar and homogeneous • Avoidance of Extreme Discontinuities: Increasing the numbers of groupings will create a more continuous spectrum of rates which will avoid discontinuities better.	
EXAMINER'S REPORT	
Candidates were expected to demonstrate knowledge of clustering methods with respect to risk classification.	

A common mistake was not being able to comment specifically on hierarchical and non-hierarchical clustering.

Part a

Candidates were expected to respond with one of the following concepts:

- Clustering creates homogeneous and credible groupings for a risk classification system; or
- Clustering will maximize the variance between classes and minimize the variance within classes.

A common mistake was mentioning homogeneity but failing to mention credibility.

Part b

Candidates were expected to have a basic understanding on hierarchical and non-hierarchical clustering.

The key concept in this part was that hierarchical clustering creates a hierarchy of clusters. This hierarchy is created in either a bottom-up fashion by grouping together two existing clusters to create a new cluster, or in a top-down fashion by breaking an existing cluster into two sub-clusters. Non-hierarchical clustering is not subject to this parent-child constraint.

Common mistakes include:

- Many candidates described hierarchical clustering by referring to an "order" in the data. These responses did not receive credit without specifically referencing the parent-child relationship described above.
- Some candidates reversed the descriptions of hierarchical and non-hierarchical clustering.

Part c

Candidates were expected to choose two considerations for risk classification and discuss how clustering methods would affect the considerations.

Common mistakes include:

- Providing similar responses for both considerations.
- Instead of providing considerations that apply to using clustering to determine new groupings, many candidates provided considerations involving the rating variable to be clustered. The question states that we are increasing the number of groupings of an existing class. For example, we are producing new groups on an existing variable, not introducing a new variable, so the "public acceptability" consideration doesn't apply, since the variable has already been accepted.

QUESTION: 3	
TOTAL POINT VALUE: 2	LEARNING OBJECTIVE(S): A1

SAMPLE ANSWERS

Part a: 1 point

Sample Responses to considerations supporting inclusion

- Statistical critieria: Credibility -> each classification is likely large enough to produce credible statistical predictions. I.e., enough policies sold through each distribution channel.
- Absence of Ambiguity -> each classification is easy to determine and likely to be mutually exclusive.
- Using distribution channels improve prediction accuracy of the expected loss of the insured.
- Since more policies can be priced more accurately, availability of coverage will increase.
- The consumer has a choice to either go to an independent agent and thus can control this selection.
- The distribution channel is easily measured and objective such that it is either one or the other.

Part b: 1 point

Sample Responses to considerations against inclusion

- Manipulation -> easily manipulated by insured (Change distribution channel based on what produces preferential pricing)
- Public Acceptability -> unclear how distribution channel is related to the insured's loss potential. No clear cause and effect relationship, not clearly based on relevant data.
- Hazard Reduction Incentive – Varying rates by distribution channel in no way promotes insureds to mitigate their hazard exposure because distribution channel is not directly linked to losses.
- Using distribution channels is more prone to insured's manipulation. They can price through different channels and select the lowest price.
- It is hard to justify for the causality to the DOI regulator to make the variable acceptable.
- Distribution channel does not necessarily reflect differences in expected loss. No reason to believe driving behavior is different and so causality does not appear to be here.
- A consumer one year could go to an agent and then the next year go online so not constancy in measure.

EXAMINER'S REPORT

Candidates were expected to identify and describe two considerations for and two against using distribution channel as a variable in coming up with their pure premium factors.

Common mistakes include:

- Describing or identifying, but not both. If a candidate described a consideration and included the key word (e.g., statistical, homogeneity, credibility, predictive stability), they got credit for identifying as well. If the key word was not included, they got the credit for describing, but not for identifying. Graders were fairly liberal in helping to identify a consideration that could fit the description given by the candidate.
- Identifying a consideration that was not one of the AAA's recommendations did not get credit for that identification, but the description would fit into another consideration, so the candidate would get credit for the description but not the identification.

- Identifying expenses as a consideration was given no credit, as the question stated that the expenses were already taken into account.
- Identifying a consideration in a (supporting inclusion) but were really more appropriate for b (against inclusion), or vice versa, were given no credit. For example, manipulation was a reason against having distribution channel as part of the rating plan, but it would not be a good example of why it should be included.

QUESTION: 4	
TOTAL POINT VALUE: 3	**LEARNING OBJECTIVE(S): A3**
SAMPLE ANSWERS	
Part a: 1.5 points	

Sample Responses for [i]

- Including limit of liability in the GLM can lead to counterintuitive results such as lower relativity for higher limit due to correlation with other variables not included in the model.
- Including limit may give unexpected results like lower rate for more coverage due to adverse or favorable selection.

Sample Responses for [ii]

- The information will not be available for new business since we are building a GLM for the prospective period.
- Number of coverage changes is likely to change from what it is in the current policy period and thereafter year by year.

Sample Responses for [iii]

- Too many ZIP codes to include it in the GLM; using a spatial smoothing technique would be more appropriate and include the determined value for ZIP code as an offset term in the GLM.
- Sparse data creates credibility concerns and it will add too many degrees of freedom to the model.
- There are too many ZIP codes to be used in a GLM. Furthermore, aggregating them into groups will cause a great loss of information.
- Too many ZIP codes create too many parameters which will potentially lead to overfitting.

Part b: 1 point

- Deductibles should lower frequency (small losses below deductible not reported) but increase severity (since claims that do get reported are higher average cost). This violates the assumption for Tweedie that variables move frequency and severity in the same direction.
- Deductible factors may produce higher relativities at higher deductibles due to factors other than pure losses elimination:
  1. Insureds at high loss potential and high premiums may elect high deductibles to reduce premium
  2. Underwriters may force high deductibles on high risks
- Deductible factors are likely correlated with other factors outside of the model and may give non intuitive results like paying more for less coverage; for example because underwriters force high risk insureds to purchase higher deductibles.

Part c: 0.5 point

- The deductible relativities can be calculated using a mix of experience and exposure rating and then included in the GLM model as an offset.
- Determine deductibles relativities by means loss elimination calculation with historical data [i.e., portion of loss not paid because of deductible E(x;d)/E(x)]. Include the relativities as an offset term in the GLM.

- Deductible relativities should be determined based purely of loss elimination, outside of the GLM model. Then they should be included as offset factors in the log-link function as +ln(relativity).

EXAMINER'S REPORT

Part a

Candidates were expected to recognize the limit of liability was a coverage option, and state a reason to not include them as well as an explanation.

Candidates did not recognize that the **current** policy period was not applicable within the **prospective** GLM and instead provided answers too general for what the question was asking for.

A common mistake was not providing a robust enough argument.

Part b

Candidates were expected to discuss why deductibles should not be used as a predictor in the GLM analysis. When candidates stated deductibles are a bad choice for a predictor variable they then had to state a reason to not include them as well as an explanation to receive full credit.

Some candidates tried to name features of the Tweedie or log-link distribution (such as a point mass at zero), but that does produce an argument against using deductibles in this case. For example, the Tweedie does well to model point masses at zero.

A common mistake was related to the Tweedie error distribution. Candidates often did not give a full explanation of the effect that deductibles would have on frequency and severity and relate that back to assumptions regarding the Tweedie.

Part c

Candidates were expected to explain how to calculate deductible relativities in general (e.g. Loss Elimination Ratios). Candidates were also expected to know how to include these calculated deductible relativities in a GLM; namely, as an offset.

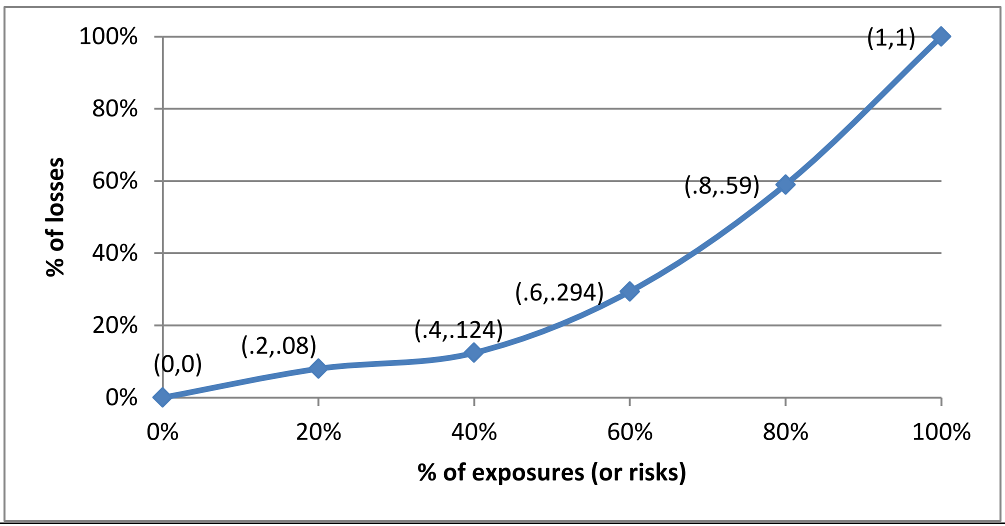
A common mistake was an insufficient or incomplete explanation of the offset procedure to include the deductible relativities in the GLM model.

QUESTION: 5	
TOTAL POINT VALUE: 2.25	LEARNING OBJECTIVE(S): A4
SAMPLE ANSWERS	

Part a: 1.75 points

Sample 1

Order	Risk	Model Predicted Loss	% Exposure	Actual Loss	% Actual Loss
1	5	200	0.2	400	0.08 (=400/5000)
2	2	500	0.4	220	0.124 (=620/5000)
3	4	800	0.6	850	0.294
4	3	1500	0.8	1480	0.59
5	1	2000	1.0	2050	1.00
Total		5000		5000	



Part b: 0.5 point

- The Gini index is twice the area between the Lorenz curve and the line of equality, where the line of equality is the straight line from the point (0,0) and (1,1) and represents where each exposure has the same expected loss.
  The Gini index measures the ability of the rating plan to differentiate between the best and worst risks, i.e. lift of the insurance plan.
- Gini index is calculated as 2 times the area between the diagonal line (line of equality) and Lorenz curve.
  In insurance rating, large Gini index represents stronger risk classification power.
- Gini index is calculated as 2 times the area between the diagonal line (line of equality) and Lorenz curve.
  It is a measure of the predictive lift of the rating program. Higher Gini index equals more lift and more predictive power.

EXAMINER'S REPORT

Candidates were expected to draw a Lorenz curves given the data, and to know how to calculate and explain the purpose of the Gini index.

Part a

Candidates were expected to calculate cumulative losses and exposures, then plot the Lorenz curve. Candidates were not expected to draw the plot to scale.

Common errors made by candidates were:

- Either not re-ordering the risks, or reordering them incorrectly.
- Using modeled losses on the Lorenz curve as opposed to actual losses.
- Graphing modeled and actual losses, without showing exposure.
- Plotting residuals as opposed to cumulative actual losses.
- Not using the cumulative percentage of loss, instead showing the actual loss for each risk.
- Calculating the percentage of loss off the highest risk instead of the total loss.

Part b

Candidates were expected to describe the Gini index calculation and what it measures.

Common errors made by candidates were:

- Missing the "2 times" the area between the Lorenz curve and line of equality.
- Stating that the Gini index measures how well the predicted losses fit the actual losses.
- Stating that the smaller the index the better the rating plan.
- Stating unclear descriptions.

QUESTION: 6	
TOTAL POINT VALUE: 2.5	**LEARNING OBJECTIVE(S): A3, A4**
SAMPLE ANSWERS	
Part a: 0.75 point	
Sample 1 (using natural log) ln(μ) = 0.910 + 4 * (0.013) + ln(25,000) * (-0.187) + 8 * 0.062 = -0.43568 μ = $e^{-.43567}$ = 64.7% *Sample 2 (using log base 10)* ln(μ) = 0.910 + 4 * (0.013) + log(25,000) * (-0.187) + 8 * 0.062 = .63559 μ = $e^{.63559}$ = 188.8%	
Part b: 0.75 point	
Sample 1 The result from the new business model can be added into the renewal model as an offset. The resulting equation is: g(μ) = ln(μ) = $\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4$ β_0 = Intercept, to be re-estimated β_1= parameter for average building age = 0.013 β_2 = parameter for log(manual premium) = -0.187 β_3 = parameter for location count = 0.062 β_4 = parameter for prior year claim count, to be estimated	
Part c: 1 point	
• **Cross-Validation** – Split the data into k parts and run the model on the (k-1) parts, then validate the result on the last part. Compare how similar the estimates are from the k iterations to assess variable stability. • **Bootstrapping** – Create multiple datasets from the initial train dataset by sampling with replacement. Run the model (with same specs) of each sampled dataset. Assess stability of estimates of coefficients by comparing the results from each run. You can compute standard errors, means and confidence intervals for the variable. • **Cook's Distance** – Sort the observations based on their Cook's Distance value (higher distance = more influence on the model.) Remove some of the most influential observations and re-run the model on this new set of data to see the effect on estimated parameters. • **Validation on Holdout Dataset** – Split the data into train and test. Run the model on the train and validate on test dataset by comparing variable fit. The models should produce similar results. • **Time- Consistency** – Review the results of model by accident year, to assess stability over time.	

EXAMINER'S REPORT
Candidates were expected to know the components of a GLM formula, GLM technical foundation and model refinement to get full credit for this question. Generally, candidates understood the components of a GLM formula, but struggled with the technical foundation of how to offset a model and with the difference between variable significance and variable stability.
Part a
The candidate was expected to use the components of the GLM formula to produce the modeled loss ratio. Both the log and ln of the manual premium were accepted as correct answers. Common mistakes include: • Using the natural log of average building age or location count. • Not converting to a loss ratio. • Using the incorrect link function.
Part b
Candidates were expected to produce the renewal business loss ratio, while keeping the coefficients from the new business model the same. It was important to recognize that the new business model was also modeling a loss ratio, not whether a policy would renew. Candidates that gave the correct formula but with no description of offset or how to apply it were not given full credit. Common mistakes include: • Describing how to model a new/renew indicator or probability of renewal (not the loss ratio). • Not keeping the coefficients of average building age, ln(manual premium) and location count the same between models. • Not recognizing that the intercept is different between models.
Part c
Candidates were expected to assess the stability of the new variable (parameter estimate). Common mistakes include: • Giving a definition of the technique without fully describing how the technique can be used to assess variable stability. • Giving answers that described how to assess variable significance with no tie-in to the concept of variable stability or giving answer on penalized measures of fit (e.g. AIC, BIC). Answers that described how to assess variable significance that did tie into the concept of variable stability were given full credit.

QUESTION: 7	
TOTAL POINT VALUE: 1.5	LEARNING OBJECTIVE(S): A4
SAMPLE ANSWERS	
Part a: 1 point	

Sample 1

AIC = -2LL + 2p
BIC = -2LL + plog(n)

Base Model:

AIC = -2(-750) + 2(10) = 1520
BIC = -2(-750) + 10log(1M) = 1560

New Model:

AIC = -2(-737.5) + 2(15) = 1505
BIC = -2(-737.5) + 15log(1M) = 1565

Sample 2

AIC Base = -2(-750) + 2(10) = 1520
AIC New = -2(-737.5) + 2(15) = 1505
BIC Base = -2(-750) + 10ln(1,000,000) = 1638.16
BIC New = -2(-737.5) + 15ln(1,000,000) = 1682.23

Sample 3

AIC = D+2p
AIC Base = 500 + 2(10) = 520
AIC New = 475 + 2(15) = 505
BIC = D + pln(n)
BIC Base = 500 + 10ln(1,000,000) = 638.155
BIC New = 475 + 15ln(1,000,000) = 682.23

Sample 4

AIC = Deviance + 2p
AIC Old = 500 + 2x10 = 520
AIC New = 475 + 2x15 = 505
BIC = Deviance + plog(n)
BIC Old = 500 + 10x6 = 560
BIC New = 475 + 15x6 = 565

Part b: 0.25 point

- With such a high number of data points, the BIC is over influenced by the ln(data points) term. The AIC is a preferable statistic for such a high sampling.
- AIC is generally more reliable because insurance models are typically built on very large datasets. BIC heavily penalizes for additional parameters and thus will normally recommend exclusion of additional variables.
- AIC is a better indicator because BIC penalizes more heavily and can cause predictive variables to be excluded.

Part c: 0.25 point

- AIC says to adopt since new AIC is lower. BIC says not to. Since AIC is more reliable, I conclude that the new model should be adopted.
- I select the new model because it has the lower AIC result. Deviance is not a good indicator here because adding parameters will necessarily improve deviance. BIC is not appropriate due to above.
- BIC New > BIC Base.
  So do not adopt the new model.
- Given that the AIC is only slightly higher than the AIC for the new model and that the BIC is lower than the BIC for the new model, I would not recommend to adopt the new model.

EXAMINER'S REPORT

Candidates were expected to calculate the AIC and BIC for 2 different models and then use these results to evaluate which model performed better.

Part a

Candidates were expected to calculate AIC and BIC for both the Base Model and the New Model. Full credit was given to candidates that used the correct AIC and BIC formulas.

Candidates could use either ln() or log() in the BIC formula and receive full credit. Deviance could be used to replace -2xLL in both formulas.

Common mistakes include:

- Using an incorrect formula (leaving out the negative or the 2x in the -2xLL portion)
- Mixing up the given information (e.g. using Deviance instead of # of parameters)
- Only calculating one AIC and one BIC statistic, using information from both models

Part b

Candidates were expected to identify why AIC was the more reliable test statistic in this situation.

Common mistakes include:

- Identifying that AIC was the better test statistic, but giving no explanation or an inadequate explanation as to why AIC is better here.
- Identifying BIC as the more reliable test statistic.
- Candidates that identified BIC as the more reliable test statistic were not given credit. Page 63 of the GLM paper clearly states that AIC is better on large datasets.

Part c

Candidates were expected to make a recommendation about which model to adopt and justify their selection.

Full credit was given for recommending either model, as long as it was supported by the AIC or BIC statistics as to why the model was chosen.

Common mistakes include:

- Giving no justification for why the New or Base model was chosen
- Giving no recommendation for which model should be used.
- Saying a higher AIC or BIC was better.
- Correctly identifying that lower AIC or BIC was better, but the conclusion listed in c) didn't match the calculations in part a)
- Using only decreasing Deviance as a reason to adopt the New model. Since Deviance always reduces when new parameters are added, it is not an appropriate metric to use (in isolation) to justify adopting a new model.
- Claiming that the statistics didn't decrease enough to justify the additional parameters. This misinterprets the statistics as they both already penalize for the additional parameters.

QUESTION: 8	
TOTAL POINT VALUE: 2.75	LEARNING OBJECTIVE(S): B1

SAMPLE ANSWERS

Part a: 1.25 points

Sample 1

With anti-selection ILF:	No anti-selection ILF:
\$25K = (6,500 / 4,000) = 1.625	\$25K = (6,500+8,000) / (4,000+6,000) = 1.45
\$50K = (9,000 / 4,000) = 2.25	\$50K = (9,000+10,500) / (4,000+6,000) = 1.95

We can see the ILFs with anti-selection are different than without anti-selection.

Sample 2

Limit = \$25K	Limit = \$50K
I(\$10K) = 1	I(\$10K) = 1
I(\$25K) = (6,500 / 4,000) = 1.625	I(\$25K) = (8,000 / 6,000) = 1.333
I(\$50K) = 2.25	I(\$50K) = 1.75

The ILF under two policy limits is significantly different. This shows anti-selection impacts the ILF. If no anti-selection, should be equal.

Sample 3

I(\$10K) = (6,000 / 4,000) = 1.5
I(\$25K) = (8,000 / 6,500) = 1.23
I(\$50K) = (10,500 / 9,000) = 1.17

When the severity limitation increases, ILF decreases, so there is anti-selection.

Part b: 1.5 points

Sample 1

Adverse Selection: Higher risk insureds choose higher policy limits
Possible reason: aware of their own riskiness, choose high limits to protect themselves
Favorable Selection: Safer insureds choose higher limits
Reason: safer risks are likely more financially stable, more able to afford higher limits

Sample 2

Adverse Selection
- Higher limits generate higher ILFs
- The liability lawsuit and settlement may be impacted by the size of the limit

Favorable Selection
- Higher limits generate lower ILFs
- Some large sized insured are good risks, they choose high limit because they have more assets to protect

Sample 3

Adverse Selection
- This is when worse than average insureds purchase higher policy limits, so worse than average loss experience is seen on the higher ILFs

- Example – insureds who expect to need high limits because they have a lot of large losses purchase high policy limits

Favorable Selection

- This is when better than average insureds select higher policy limits, so better than average loss experience is observed for higher ILFs
- Example – underwriting is willing to give good insureds higher policy limits

EXAMINER'S REPORT

Part a

Candidates were expected to understand anti-selection, and that the presence of it results in different ILFs between the total population and the group. They were expected to calculate the ILFs with and without anti-selection, and conclude whether anti-selection exists.

Common mistakes include:

- Concluding that there is anti-selection because the limited severities differed between policy limits $25K and $50K.
- Testing for ILF consistency to determine whether there is anti-selection. This is the wrong test as the consistency test will not always fail if there is anti-selection.

Part b

Candidates were expected to identify two different types of anti-selection: Adverse/Negative/Unfavorable/etc., and Favorable/Positive/Beneficial/etc. They were also expected to describe a relationship between high limit policies and good/bad loss experience.

Common mistakes include:

- Not describing what adverse or favorable anti-selection was, but only giving an example (e.g. court settlements are influenced by policy limit – this does not give any information on performance of high limits).
- Giving a general description of Adverse Selection which was not specific to impact on ILF (e.g. mispricing model that attracts more high risk insureds – this description is not specific to ILF).

QUESTION: 9	
TOTAL POINT VALUE: 1.5	LEARNING OBJECTIVE(S): B3,B4

SAMPLE ANSWERS

Part a: 0.5 point

- The underwriter should consider if the risk is a proper fit for its classification, especially since the class experience has improved, while this risk's experience shows deterioration. However, it is also possible that the recent experience is a random fluctuation. If this risk does seem to belong within this class code, it may foretell good experience if randomness goes the other way. So, should renew.
- Exp Mod ↑ ≠ bad risk!
  ELR on class code ↓ consistently means that class hasn't been rated appropriately in the past
  → ELRs down b/c exp mods compensating for inadequate rates
  Most recent year not in exp mod
  Do NOT non-renew, wait for latest year of experience then re-assess
- If the class code ELR has been decreasing and the experience mod has been increasing, it would be worth examining the classification of the risk to see if it would fit better in a different class. Regardless, the experience mod is keeping the risk's rate adequate and it should not be non-renewed.
- The experience modification should effectively adjust for the individual's expected cost difference from the average of the class. The underwriter can check the modified loss ratios of this insured for the past few years. If experience modification is appropriate and equitable rates are charged, modified loss ratios should be similar for all credit and debit risks that are in the company's book of business. If this is true, then there is no reason for the underwriter not to renew the risk.

Part b: 0.5 point

- Not supposed to use current premium to recoup losses, only to collect what we expect to pay in the future – bad reasoning
- Ratemaking is not intended to recoup for past losses. The increased experience mod reflects increased knowledge about expected losses in the prospective period. Despite incorrect reasoning the policy should still be renewed.
- The debit modification is to ensure all modified loss ratios of both credit and debit risks are similar. Hence all debit and credit risks are equally profitable. It is wrong to say that a debit mod helps recoup losses as all other risks are contributing the same profit. The underwriter should renew the account knowing that adequate rates are charged for expected future losses.

Part c: 0.5 point

- Small risks that have poor loss experience may not qualify for experience rating, or may receive little credibility. Those small risks that would have had mods >1 now have mods close to 1 in the off-balance calculation, which decreases the average mod.
- The underlying accounts may pay higher than average wages to their workers, which will generate more expected losses. These higher expected losses will decrease the emods, possibly generating a net credit emod.
- Larger risks tend to have better experience, resulting in credit mods that get a lot of weight in the book. The book could have lots of small policies not eligible for experience rating that have horrible experience.

- Larger risks meet criteria for experience rating, and they tend to be able to afford better safety programs and are more likely to have credit mods.
- It could be the case that manual rates have been set too high, and the experience mod is correcting for this.
- The mod is not a predictor of plan performance. After the application of the mod, all risks should be equal.
- Selecting risks with little prior losses and experience credits does not mean that these risks will outperform in the future.

EXAMINER'S REPORT

Candidates were expected to show basic understanding of the use and purpose of experience rating in Workers Compensation.

Part a

Candidates were expected to identify the potential for poor class fit and explain that the experience mod is intended to correct for this.

Graders also accepted what that the increasing experience mod could be due to random large losses, if the candidate explained their response.

Only stating the mathematical reasoning for the experience mod increasing and not discuss why this would be taking place did not receive full credit.

A common mistake was connecting the decreasing ELRs with rate adequacy. While an increasing experience mod can result from deteriorating rate adequacy, this does not necessarily translate to decreasing ELRs.

Part b

Candidates were expected to evaluate the underwriter's assertion regarding using an experience mod to recoup prior losses. Candidates needed to identify that the rationale is incorrect, as pricing is prospective and does not recoup prior losses.

A common mistake was discussing the random nature of losses, low credibility or capping in the experience mod calculation. These responses were not awarded credit as they did not evaluate the underwriter's reasoning (experience mod allows for recouping of prior losses).

Part c

Candidates were expected to provide two separate reasons as to why an overall credit mod doesn't indicate superior risk selection. Many different reasons were accepted, as long as they were accurate.

Common mistakes include:

- Stating that the mod should be less than 1.0 because a) large risks tend to have better experience and b) large risks are more credible. This was graded as one reason as these comments are not sufficient on their own.
- Stating that small risks are less credible and thus the experience mods are closer to 1.0. This did not address the question of why the overall mod would be less than 1.0. Unless further reasoning was provided, this answer was not accepted.

QUESTION: 10	
TOTAL POINT VALUE: 2.5	LEARNING OBJECTIVE(S): B3
SAMPLE ANSWERS	

Sample 1

Use 2011 tables

Lookup Class 7705: ELR = 2.02, D = 0.17

$$E = \frac{5{,}000{,}000}{100} * 2.02 = 101{,}000$$

$$E_E = 101{,}000 * (1 - 0.17) = 83{,}830$$

Lookup E: W = 0.14, B = 28,000

Claim	A_p	A_e
1	5,000	24,000=29,000-5,000
2	1,500=5,000*0.3	7,650=(30,500-5,000)*0.3
3	5,000	85,000
4	1,500	0
5	1,500	12,000=(45,000-5,000)*0.3
Total	14,500	128,650

$$\begin{aligned} Mod &= \frac{A_p + wA_e + (1-w)E_e + B}{E + B} \\ &= \frac{14{,}500 + 0.14 * 128{,}650 + (1 - 0.14) * 83{,}830 + 28{,}000}{101{,}000 + 28{,}000} \\ &= 1.03 \end{aligned}$$

Sample 2

Use 2010 tables

Class 7705: ELR = 1.84, D-ratio = 0.2

$$Expected\ Loss = \frac{5{,}000{,}000}{100} * 1.84 = 92{,}000$$

W = 0.14, B = 26,800

$$E_E = 92{,}000 * (1 - 0.2) = 73{,}600$$

Risk	A_p	A_e
1	5,000	24,000=29,000-5,000
2	1,500=5,000*0.3	7,650=(30,500-5,000)*0.3
3	5,000	85,000
4	1,500	0
5	1,500	12,000=(45,000-5,000)*0.3
Total	14,500	128,650

$$Mod = \frac{14{,}500 + 0.14 * 128{,}650 + (1 - 0.14) * 73.600 + 26{,}800}{92{,}000 + 26{,}800}$$
$$= 1.03$$

Sample 3
Use 2011 tables
Based on Class Code 7705: ELR = 2.02, D = 0.17

$$Expected\ Loss = \frac{5{,}000{,}000}{100} * 2.02 = 101{,}000$$
$$E_P = 101{,}000 * 0.17 = 17{,}170$$
$$E_E = 101{,}000 - 17{,}170 = 83{,}830$$

Claim	A_p	A_e
1	5,000	24,000=29,000-5,000
2	1,500=5,000*0.3	7,650=(30,500-5,000)*0.3
3	5,000	85,000
4	1,500	0
5	1,500	12,000=(45,000-5,000)*0.3
Total	14,500	128,650

Based on Expected Loss: W = 0.14, B = 28,000

$$Z_p = \frac{E}{E + B} = 78.29\%$$

$$Z_e = w * Z_p = 10.96\%$$

$$Mod = \frac{A_p * Z_p + E_p * (1 - Z_p) + A_e * Z_e + E_e * (1 - Z_e)}{E}$$
$$= \frac{103{,}821.929}{101{,}000}$$
$$= 1.03$$

EXAMINER'S REPORT

Candidates were expected to use the NCCI manual to correctly calculate an experience modification factor.

Credit was given for using either the 2010 or 2011 tables, as long as they were used consistently.

Common mistakes include:
- Incorrectly calculating ratable losses for medical only claims, i.e., not applying 0.3 factor to medical only claims.
- Mixing values from different years, e.g. selecting ELR from 2011 table but picking up W and/or B from 2010 or vice versa.
- Incorrectly selecting W and/or B from the tables.

QUESTION: 11	
TOTAL POINT VALUE: 3.75	LEARNING OBJECTIVE(S): B4,A1
SAMPLE ANSWERS	
Part a: 3 points	

Sample 1:

Group risks into quintiles, ordered by Modification.

Quintile	Current	Proposed
1	1,3	1,2
2	2,5	3,4
3	4,7	5,6
4	6,9	7,8
5	8,10	9,10

Calculate Manual and Standard Loss Ratios by group.

	Manual	
Quintile	Current	Proposed
1	0.60	0.55
2	0.75	0.75
3	1.00	1.00
4	1.35	1.35
5	1.65	1.70

Standard	
Current	Proposed
0.89	1.05
0.91	1.03
0.95	1.00
1.15	0.98
1.29	0.91

The manual loss ratios under the proposed plan are more dispersed than under the curren
Therefore, the proposed plan is better at identifying risk differences.
The standard loss ratios under the proposed plan are closer to 1 (or show less variance) th
the current plan. Therefore, the proposed plan is better when comparing standard loss rat
Overall, the proposed plan is better. However, the proposed plan does show a decreasing
standard loss ratio which suggests that the proposed plan puts too much credibility on exp

Sample 2:

Current Plan

Risk	Man LR	Std LR
1	50%	76.9%
3	70%	100.0%
2	60%	80.0%
5	90%	100.0%
4	80%	80.0%
7	120%	109.1%
6	110%	95.7%
9	160%	133.3%
8	150%	120.0%
10	180%	138.5%

Proposed Plan

Risk	Man LR	Std LR
1	50%	100.0%
2	55%	109.1%
3	70%	100.0%
4	75%	106.7%
5	95%	94.7%
6	105%	104.8%
7	120%	96.0%
8	150%	100.0%
9	160%	91.4%
10	180%	90.0%

In the current plan, the manual loss ratios are not monotonically increasing which is a problem. The proposed plan does not have this, therefore it is the better plan. Looking at the standard loss ratios, the proposed plan is generally closer to 1, while the current plan shows much more variation from 1, therefore the proposed plan corrects for differences in manual loss ratio better. The current plan has a decreasing trend in standard loss ratios which means the plan may be assigning too little credibility, while the proposed plan may be assigning too much credibility as can be seen by the decreasing trend in standard loss ratios.

Sample 3:

Rank by Current Mod

Risks	Man Prem	Losses	Man LR	Std LR
1, 3	2,000	1,200	60%	88.9%
2, 5	2,000	1,500	75%	90.9%
4, 7	2,000	2,000	100%	95.2%
6, 9	2,000	2,700	135%	114.9%
8, 10	2,000	3,300	165%	129.4%
		Variance	0.1486	0.0248
Test Stat = Var(Std LR) / Var(Man LR) =				0.1670

Rank by Proposed Mod

Risks	Man Prem	Losses	Man LR	Std LR
1, 2	2,000	1,100	55%	104.8%
3, 4	2,000	1,500	75%	103.4%
5, 6	2,000	2,000	100%	100.0%
7, 8	2,000	2,700	135%	98.2%
9, 10	2,000	3,400	170%	90.7%
		Variance	0.1706	0.0025
Test Stat = Var(Std LR) / Var(Man LR) =				0.0144

Based on the efficiency test the proposed plan has a lower test statistic therefore it is the better plan.
The proposed plan does have a downward trend in the standard loss ratio indicating it is giving too much credibility. The current plan has the opposite problem. Based on this the proposed plan is still superior.

Sample 4:

Using the Meyers Efficiency Test, choose the plan with the lowest test statistic, where the test statistic is defined as:
Test Statistic = Variance (Modified Loss Ratios) / Variance (Manual Loss Ratios)
Current Plan:

Risk	Manul Prem	Losses	Man LR	Mod	Std LR
1	1,000	500	50%	0.65	76.9%
3	1,000	700	70%	0.70	100.0%
2	1,000	600	60%	0.75	80.0%
5	1,000	900	90%	0.90	100.0%
4	1,000	800	80%	1.00	80.0%
7	1,000	1,200	120%	1.10	109.1%
6	1,000	1,100	110%	1.15	95.7%
9	1,000	1,600	160%	1.20	133.3%
8	1,000	1,500	150%	1.25	120.0%
10	1,000	1,800	180%	1.30	138.5%

Variance (Std LRs)	0.0431
Variance (Man LRs)	0.1801
Test Statistic	0.2395

Proposed Plan:

Risk	Manul Prem	Losses	Mod	ManLR	Std LR
1	1,000	500	0.50	50%	100.0%
2	1,000	600	0.55	60%	109.1%
3	1,000	700	0.70	70%	100.0%
4	1,000	800	0.75	80%	106.7%
5	1,000	900	0.95	90%	94.7%
6	1,000	1,100	1.05	110%	104.8%
7	1,000	1,200	1.25	120%	96.0%
8	1,000	1,500	1.50	150%	100.0%
9	1,000	1,600	1.75	160%	91.4%
10	1,000	1,800	2.00	180%	90.0%

Variance (Std LRs)	0.0036
Variance (Man LRs)	0.1801
Test Statistic	0.0201

By the efficiency test, the proposed plan has the lower test statistic, therefore this plan is preferred over the current plan. Note that we could have also looked solely at the variance of the modified loss ratios, as the denominator in the test statistics is identical.

Looking at the standard loss ratios when the risks are ranked by the mods, we can see a clear increasing trend in the standard loss ratios in the current plan. This implies the current plan is not assigning enough credibility to the actual risk experience. Looking at the proposed plan, the trend is not as pronounced, but there is a small decreasing trend in the standard loss ratios when ranked by the proposed mods. This implies the proposed plan is assigning too much credibility to the actual risk experience.

Part b: 0.75 point

- Robots average loss = 800, Made by hand average loss = 1340. It is apparent that the average loss for made by hand is higher than robot. The plan is not doing a good job in differentiating the loss potential between the two types of manufacturing. The made by hand class should be charged a higher manual rate.
- Risks that use robots consistently have lower mods than made by hand risks. Class may not be granular enough. Should consider splitting into two classes by manufacturing type if there is enough credibility to have two smaller classes.

EXAMINER'S REPORT

Candidates were expected to demonstrate knowledge of classification plans and experience rating plans and how to evaluate different plans.

C Common mistakes include:

- Misunderstanding the class structure
- Not providing full evaluations of the experience rating plan and/or classification plan.

Part a

Candidates were expected to demonstrate the ability to compare two experience rating plans.

A number of approaches were allowed for full credit including the efficiency test and quintiles test.

Common mistakes include:

- Not assigning risks to the correct quintile.
- Not addressing the trend in standard loss ratios that suggests too much/little credibility in the proposed and current plans respectively.
- Grouping by manufacturing type. Syllabus readings recommend separating tests by premium size but otherwise never mention separating within a single class. The question specifically stated that all risks were part of a single manufacturing class, therefore, candidates lost some credit for separating the 10 risks by manufacturing type in efficiency and quintiles tests.

Part b

Candidates were expected to evaluate the appropriateness of these ten risks being grouped together in a single class.

Full credit answers recognized the class is not homogenous and demonstrated this by calculating the manual loss ratios, average loss, or experience mods for Robots and Made by hand manufacturing types.

Candidates also received credit for addressing credibility concerns with further refining the class.

Candidates who included a response in Part b that pertained to Part a, such as discussing the trend in proposed standard loss ratios and the implication of too much credibility, were given the appropriate credit in Part a.

Common mistakes include:

- Not discussing class fit.
- Not recognizing that the risks are currently part of the same class.
- Not fully justifying the decision to separate Robots and Made by hand into two classes.

QUESTION: 12	
TOTAL POINT VALUE: 2.5	LEARNING OBJECTIVE(S): B2,B5
SAMPLE ANSWERS	
Part a: 1.5 points	

Sample
E = 85k

Risk	r = L/E
1	1
2	1.5
3	0.5
4	0.75
5	1.25

Entry Ratio	n_i	$n_i > r_i$	$\% > r_i$	ϕ	ψ
0	0	5	1	1	0
0.25	0	5	1	0.75	0
0.5	1	4	0.8	0.5	0
0.75	1	3	0.6	0.3	0.05
1	1	2	0.4	0.15	0.15
1.25	1	1	0.2	0.05	0.3
1.5	1	0	0	0	0.5

$\phi(r_{max}) = 0$

$\phi(r_i) = \phi(r_{i+1}) + (r_{i+1} - r_i)(\% > r_i)$

$\psi(r) = \phi(r) + r - 1$

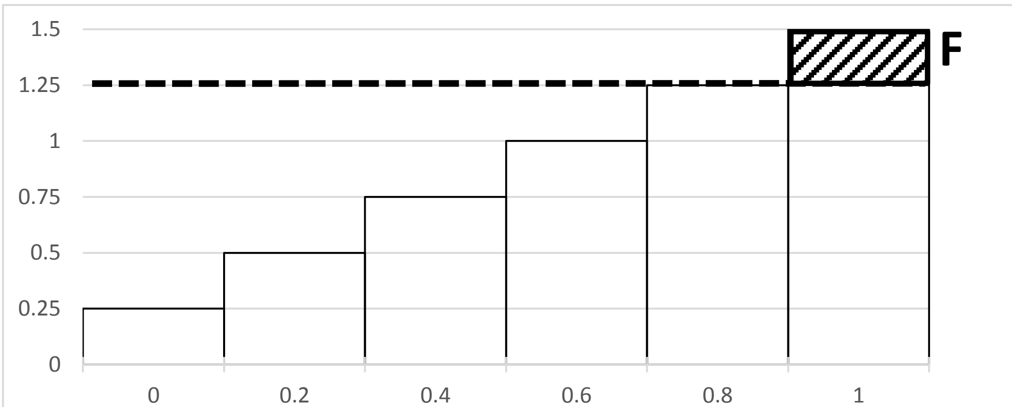
Sample 2
Average loss = (85k + 127.5k + 42.5k + 63.75k + 106.25k) / 5 = 85000
Entry ratio 1.5 is equivalent to 1.5 x 85,000 = 127,500, the highest loss

Entry Ratio	# of claims above	Incremental Charge	ϕ	ψ
0	5	5/5 x 0.25 = 0.25	1	0
0.25	5	5/5 x (0.5 – 0.25) = 0.25	0.75	0
0.5	4	4/5 x (0.75 – 0.5) = 0.2	0.5	0.5 – 1 + 0.5 = 0
0.75	3	3/5 x (1 – 0.75) = 0.15	0.3	0.3 – 1 + 0.75 = 0.05
1	2	2/5 x (1.25 – 1) = 0.1	0.15	0.15 – 1 + 1 = 0.15
1.25	1	1/5 x (1.5 – 1.25) = 0.05	0.05	0.05 – 1 + 1.25 = 0.3
1.5	0	0	0	0 – 1 + 1.5 = 0.5

Part b: 0.25 point

Sample 1

$\phi(1.25)$ = shaded area/total area under F



Sample 2

$\phi(1.25)$ reflects the excess portion of expected losses that are above 1.25 times the mean. In this example it is the portion of losses that are greater than 1.25 x 85k = 106,250. It would be the expected sum of the excess portion of losses > 106,250 divided by the total mean.

Sample3

1.25(85k) = 106,250

$\phi(1.25)$ represents the average amount by which the aggregate losses exceed 106.25k as a % of expected total losses.

Part c: 0.75 point

Sample 1

Actual Avg Loss

1/5(12k + 42.5k + 63.75k + 106.25k + 275k) = 99.9k

1. The table above is inappropriate to use. The expected losses based off the actual losses were 99,900 whereas the expected losses in the table we calculated above were 85,000. Typically larger risks have less variance in their entry ratios and therefore have flatter curves. The curve is likely inappropriate, as a flatter curve will result in smaller insurance charges. We would be overstating the charge if we were to use the table above.
2. Only one of the actual losses is above the entry ratio of 1.25 whereas before 2 were.

risk	Entry ratio
1 12k	.12
2 42.5k	.425
3 63.75k	.6376
4 106.25k	1.06
5 275k	2.75

Sample 2

The expected LR is much higher (99k vs. 85k) and there is more volatility in the losses. Using Table M above will understate the charge and understate the savings. This could lead to a net insurance charge that is higher or lower than actual.

Sample 3

E(X) = 99,900 r_1 = 12k/99.9k = .120 r_2 = .4254 r_3 = .6381 r_4 = 1.0636 r_5 = 2.7528

1. I wouldn't use the above table M because there is a wider dispersion of r in this second table compared to the first. The charge at r = 2 would be 0 for the first table and >0 in the second table if created because there was a loss greater than 2 as evidenced in r_5 = 2.7528.
2. The expected losses are different which could imply a different loss distribution so I would not use the first table. In the NCCI retro manual, you look up the expected losses to get an expected loss group because as expected losses increase, we would expect lesser variation in the losses.

EXAMINER'S REPORT

Candidates were expected to demonstrate the techniques to construct Table M, and be able to evaluate the appropriateness of using a particular Table M.

Part a

Candidates were expected to construct a Table M with insurance charges and savings using the provided loss data points.

Common mistakes include:
- Missing the insurance savings column.
- Not showing all work on how the charges/savings were being derived.

Part b

Candidates were expected to describe the definition of insurance charge.

Responses in both verbal form and drawings were accepted.

A common mistake was an insufficient explanation of the insurance charge.

Part c

Candidates were expected to demonstrate the appropriateness of using the table M, constructed in Part a, for the actual losses provided in Part c, with two supporting justifications.

Most candidates correctly stated the Table M, from Part a, was not appropriate to use in this case. To receive full credit though, candidates needed to be able to state in some way that the retro plan will be out of balance with an identical Guaranteed Cost book.

Common mistakes include:
- Not stating in some way that the retro plan would be out of balance with an identical guaranteed cost book.
- Stating that Table M, from Part a, lacked credibility because it was only based on 5 data points. This was not an accepted justification in this case.

QUESTION: 13	
TOTAL POINT VALUE: 2.5	**LEARNING OBJECTIVE(S): B5**
SAMPLE ANSWERS	
Part a: 1.25 points	

Sample 1

In the chart: $E = 0.2 * 0.1 + 0.4 * 0.1 + 0.6 * 0.1 + 0.8 * 0.2 + 1 * 0.2 + 1.2 * 0.1 = 0.6$

From the chart, we can observe

$$\emptyset_{r_G} = \frac{[0.2 * (1 - 0.8) + 0.1 * (1.2 - 0.8)]}{0.6} = 0.1333$$

$$\varphi_{r_H} = \frac{0.2 * 0.2}{0.6} = 0.0667$$

$$Converted\ charge = cI = 1.085 * \left(\emptyset_{r_G} - \varphi_{r_H}\right) * E$$

$$= 1.085 * (0.1333 - 0.0667) * (0.6 * 10{,}000{,}000) = \$434{,}000$$

Sample 2

$$E\emptyset_{r_G} = [0.2 * (1 - 0.8) + 0.1 * (1.2 - 0.8)] = 0.08$$

$$E\varphi_{r_H} = 0.2 * 0.2 = 0.04$$

$$Converted\ charge = cI = 1.085 * \left(E\emptyset_{r_G} - E\varphi_{r_H}\right)$$

$= 1.085 * (0.08 - 0.04) = 0.0434$ as a % of Standard Premium

\$434,000 in dollars

Part b: 1.25 points

Sample 1

$$Basic\ premium = b = e - (c - 1) * E + cI$$

$$b = (0.97 - 0.6) * 10{,}000{,}000 - (1.085 - 1) * 6{,}000{,}000 + 434{,}000 = 3{,}624{,}000$$

$$Retrospective\ premium = R = (b + c * L) * T$$

$L = \frac{8{,}700{,}000}{10{,}000{,}000} = 0.87 > 0.80$, so use 80% loss ratio at the maximum premium, i.e. 8,000,000

$$R = 12{,}500{,}000 = (3{,}624{,}000 + 1.085 * 8{,}000{,}000) * T$$

Solve for T

T = 1.0159

Sample 2

$$\emptyset_{r_H} - \emptyset_{r_G} = \frac{\left(e + E(A)\right)T - H}{E(A)T}$$

$$r_G - r_H = \frac{G - H}{E(A)T}$$

$$\emptyset_{r_H} - \emptyset_{r_G} = 0.733 - 0.133 = 0.6 = \frac{.97T - H}{1.085 * 0.6 * T}$$

$$r_G - r_H = 1.0 = \frac{1.25 - H}{1.085 * 0.6 * T}$$

$$0.5794T = 1.25 - 0.6510T$$

$$1.25 = 1.2304T$$

$$T = 1.0159$$

EXAMINER'S REPORT

Candidates were expected to understand the concepts underlying the construction of a retrospective rating plan.

Part a

Candidates were expected to calculate the insurance charge and insurance savings, and then convert that into a Net Converted Insurance Charge.

Common mistakes include:
- Not dividing the insurance charge and savings by the expected losses, E
- Failing to calculate and include the insurance savings. If a candidate did not calculate the Net insurance charge in Part A, but properly included the insurance savings as part of the derivation of Part B, full credit was given.

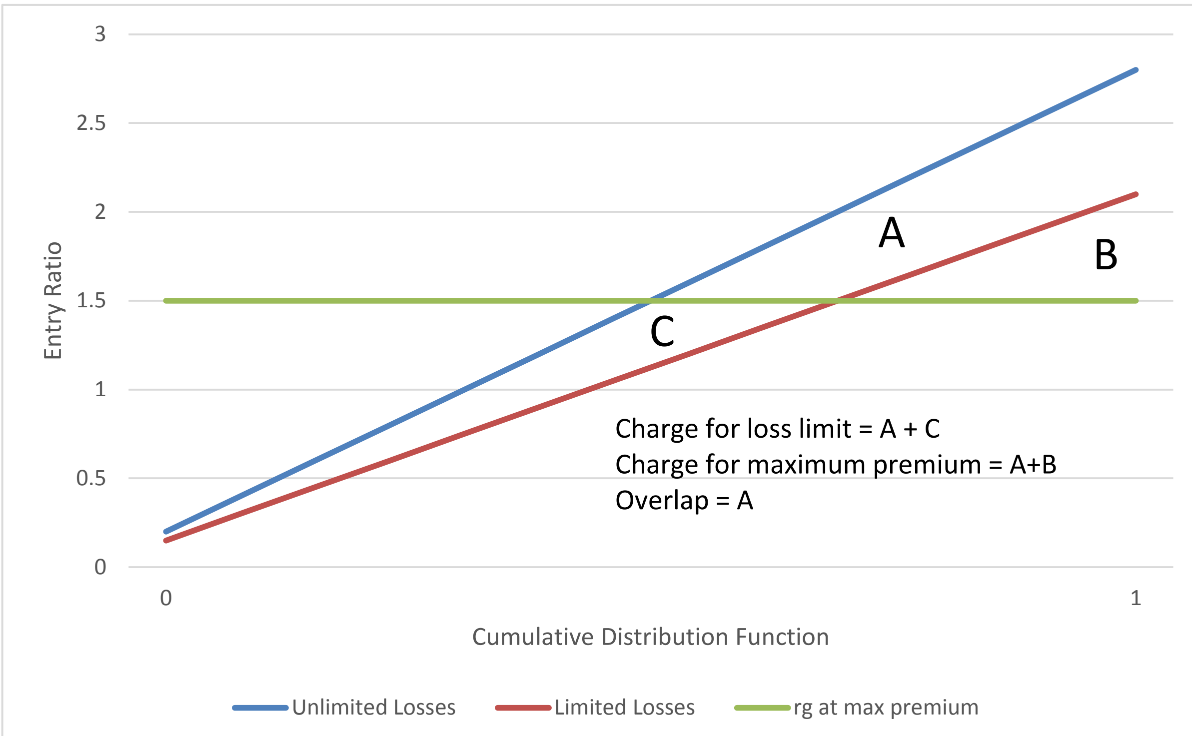
Part b

Candidates were expected to use the answer from Part a, along with the basic premium equation and retrospective premium equation to determine the tax multiplier.

Common mistakes include:
- Not recognizing that the Provision for Losses and Total Expenses Exclusive of Taxes (0.97) represents e+E
- Using the actual ultimate losses of 8,700,000 instead of the losses at the maximum premium of 8,000,000

QUESTION: 14	
TOTAL POINT VALUE: 1.75	**LEARNING OBJECTIVE(S): B5**
SAMPLE ANSWERS	
Part a: 0.5 point	

- Draw a Lee Diagram labeling unlimited loss curve, limited loss curve, entry ratio at max premium, and indicating where the charges overlap



- When there is an occurrence limit as well as an aggregate limit, the occurrence limit makes it less likely that the aggregate limit will be hit. Thus, the charge for the aggregate limit should be reduced, otherwise there will be overlap.

Part b: 0.5 point

Sample Responses for Table M

- Adjust expected loss to a larger size to approximate a limited table M
- It uses the AEL procedure to make aggregate loss distribution less skewed approximating the overlap correction
- Uses the ICRLL procedure to shift the curve to approximate a limited loss curve
- ICRLL process involves using a larger sized insured which has a flatter distribution

Sample Responses for Table L

- Charges are calculated with both an occurrence and aggregate limit
- Builds tables for separate limits on capped losses

Part c: 0.75 point

Sample Responses for Table M

- Based on countrywide data so more credible
- More easily updated for inflation by adjusting ELG table
- More flexible for changing loss limits from year to year as you do not need a separate table for each limit
- Table L is built using California taxes so not appropriate for use in other states

Sample Responses for Table L

- Table L provides more accurate estimation for the insurance charge
- Since there is a fixed loss limit there is not a need for a large number of tables to accommodate changing limits
-

EXAMINER'S REPORT

Candidates were expected to understand how charges for an individual loss limit and a maximum premium overlap, and how that is handled between Table M and Table L.

Part a

Candidates were expected to know how charges for a loss limitation and maximum premium overlap.

Common mistakes include:

- Identifying that having an individual loss limit reduces the likelihood of hitting the aggregate loss associated with the maximum premium without also commenting on how that relates back to the charges for the loss limitation and maximum premium.
- Drawing the Lee Diagram but not correctly identifying the overlap area.

Part b

Candidates were expected to know how the overlap in charges is handled differently in Table M and Table L. Candidates performed very well on this part.

Common mistakes include:

- Stating ICRLL without providing a brief description of the process.
- Stating that Table L does not have an overlap.

Part c

Candidates were expected to know differences between Table M and Table L which could support a recommendation to use one table versus the other.

Common mistakes include:

- Assuming that part C was a policy with only a loss limit and not a maximum premium. This was an invalid assumption within the context of the question as there would be no need to use a Table M or Table L without the presence of a maximum premium.
- Assuming the loss limit meant an aggregate limit and there was no per occurrence limit. The intro to the question stated this was a retrospective rated policy with a loss limitation and a maximum premium. Within this context, a loss limit only refers to a per occurrence loss limitation.
- Stating that Table L could not be used outside of California (it shouldn't but it is).
- Stating that Table L was built using California data without providing any further detail (like that the CA taxes are included in the table – which is why you shouldn't use it outside of CA).
- Claiming that Table L is easier because you don't need to add the occurrence charge separately.

QUESTION: 15	
TOTAL POINT VALUE: 1.75	LEARNING OBJECTIVE(S): B7
SAMPLE ANSWERS	
Part a: 1.25 points	

Sample 1

Avg Unlimited Loss + ALAE = (100+200+200+300+500+500+600+800+800+1000) x (0.1) = 500 (000)

Minimize Insured's Loss Retention by finding minimum deductible.
I assume that the question is asking for the aggregate deductible. Assume maximum premium of 100K is referring to the maximum premium for an excess WC policy.
Excess WC Premium = [XL x (1 + ULAE) + SP(GO)] / (1 – A – T – P)

100,000 = [XL(1.05) x 840,000 (0.02)] / [1 – 0.05 – 0.03 – (-0.088)]

XL = 80,000, or Excess Ratio = 80,000 / 500,000 = 0.16.

Find the aggregate limit such that the excess ratio = 0.16:
Try 600K:

Using the distribution provided, Excess Ratio at 600K = [800k x 2 + 1000k – 600k x (3)] / [500k x 10] = 0.16.

Therefore, the deductible is 600k.

Sample 2

Excess Premium = [EEL x (1 + ULAE) + SP x (GO)] / (1 – A – T – P)

100,000 = [EEL x (1.05) + (840,000) x (0.02)] / [1 – 0.05 – 0.03 – (-0.088)]

EEL = 80,000

Using Trial and Error, let deductible = 600,000:

EEL = (800,000 – 600,000) x (0.2) + (1,000,000 – 600,000) x (0.1) = 80,000

Therefore, the minimum retention is 600,000.

Sample 3

LDD Premium = [EX x (XS + LBA + ULAE) + SP (CR + GO)] / (1 – A – T – P)

EX = expected total loss = (100+200+200+300+500+500+600+800+800+1000) x (0.1) = 500 (000).

LDD Premium = [500 x (XS + 0.02 + 0.05) + 840 x (0.04 + 0.02)] / [1 – 0.05 – 0.03 – (-0.088)]

= 100

XS = 0.0308

Expected Excess Loss = 0.0308 (500,000) = 15,400

If deductible = 800k then EEL = 0.1 x 200,000 = 20,000
If deductible = 900k then EEL = 0.1 x 100,000 = 10,000
If deductible = 846k then EEL = 0.1 x 154,000 = 15,400.
Therefore, the deductible is 846,000.

Part b: 0.5 point

- The profit load for LDD is typically higher than XS because for XS you're competing basically just on price, but for LDD you're competing on price and service.
- Profit load in LDD tends to be higher because they are not just competing on price. They are also competing on service.
- Profit load for excess policy is generally smaller than an LDD policy. LDD policy competition is driven by both quality of service and price, since LDD provides full service. Because excess provides service only for claims above deductible, service quality is less important and price is the main concern. This drives down profits for excess policies due to competitive forces.

EXAMINER'S REPORT

Candidates were expected to apply the formula for the premium of an excess policy to solve for the expected excess loss amount that corresponds to $100,000 in premium. Then, candidates were expected to use the Lee diagram provided to determine the deductible that would result in an expected excess loss amount equal to the amount derived in the first portion of part a.

Part a

Candidates were expected to show the formula and calculations used in deriving the expected excess loss amount, and then explain the connection to the Lee diagram (either quantitatively by showing their calculations, or through words) and calculate the correct deductible that results in the expected excess loss amount derived.

The question asked for the "deductible" for an excess workers compensation policy, which caused some candidates to believe that the question was asking for the premium for an LDD policy (an excess policy has a "retention", not a deductible). Because of this, candidates did not lose credit for this approach.

Common mistakes include:

- Not making connection between using the Lee diagram to calculate the deductible amount.
- Providing an incorrect formula for the premium of an excess or LDD policy.
- Showing insufficient work for calculations.
- Incorrectly using the Lee diagram to calculate the deductible amount.

Part b

Candidates were expected to state that the profit provision for an excess policy is generally lower than that of a LDD policy. Additionally, candidates were expected to explain that the primary reason for this is that for LDD policies, insurers are able to compete on both price and service, while for excess policies insurers compete primarily on price (which drives down the profit load).

Many candidates also provided additional information such as noting that the longer average payout period for excess policies resulting in greater opportunity for investment income. While this is true, it was not required to receive credit.

Common mistakes include:

- Stating that LDD policies typically have a lower profit provision.
- Not providing a correct explanation for why excess policies have a lower profit provision.

QUESTION: 16	
TOTAL POINT VALUE: 3.75	**LEARNING OBJECTIVE(S): B6,B7**
SAMPLE ANSWERS	
Part a: 2.75 points	

Sample 1

Want to know difference in price between Option 1 and expiring.

$$\text{Price of Expiring} = \frac{(\text{XS Loss} + \phi(r) \times \text{EPL}) + \text{EL} \times (\text{ULAE} + \text{LBA}) + \text{FE}}{1 - \text{PVE}}$$

FE = 15,000
PVE = 0.17
EL = 500,000
XS loss = 500,000 x 0.4 = 200,000
ULAE = 5%
LBA = 3%
EPL = 500,000 – 200,000 = 300,000
r = 300,000 / 300,000 = 1
$\phi(r) = 0.33$

$$\text{Price} = \frac{(200{,}000 + 0.33 \times 300{,}000) + 500{,}000 \times (0.05 + 0.03) + 15{,}000}{1 - 0.17} = 426{,}506$$

Option 1:

XS Loss = 500,000 x 0.2 = 100,000
EPL = 500,000 – 100,000 = 400,000
r = 400,000 / 400,000 = 1
$\phi(r) = 0.35$

$$\text{Price} = \frac{(100{,}000 + 0.35 \times 400{,}000) + 500{,}000 \times (0.08) + 15{,}000}{1 - 0.17} = 355{,}422$$

355,422 – 426,506 = -71,084

Option 1 is $71,084 cheaper than expiring.

Sample 2

Expiring: [500 x (0.4 + 0.6 x 0.33) + 500 x (0.05 + 0.03) + 15] / (1 – 0.17) = 426.5

Option 1: [500 x (0.2 + 0.8 x 0.35) + 500 x (0.05 + 0.03) + 15] / (1 – 0.17) = 355.4

Δ = 71.1 K

Part b: 1 point

- I recommend Option 2. Option 2 does not list an aggregate retention limit and so will be more protected against losses increasing above this (in aggregate). Also, not having adjusting expenses on losses below the limit will also reduce costs. This will also protect the insurer against possible credit risk if the insured is unable to pay the unexpectedly higher losses and so can't reimburse insurer.

- Go with Option 1. You can still handle all claims, and try to keep the costs down. (Insured may not have as much incentive to keep claims from piercing above 200K). Excess workers compensation has longer average payout so more uncertain. The shorter payout for option 1 will help offset risks from trend.
- Excess Policy: Insurer can save on costs by not having to adjust claims. Also, if insured moved their deductible up, the credit risk increases for insurer → would need more collateral. This is contingent on market recognizing trend and not pursuing profit margins too low to where insurer can't make money. Since longer payout period for an excess policy (compared to LDD) should be able to make up lower profit margin through longer tailed investments (and higher return). If trend doesn't hold up but market has adjusted for it, then should make higher than expected profit. If trend turns out to be related to frequency, then better off not accumulating these attritional losses → higher risk of hitting agg if frequency increasing XS policy wouldn't have to provide additional coverage for 1st dollar claims.
- I would recommend Option 1 – The LDD policy. An upward trend in ground up losses would be amplified for excess losses for two reasons:
  - i. For losses that are already above the retention, trend will apply completely in the excess layer.
  - ii. For losses just below the retention, trend will push then above the retention, creating new excess losses.

  This means that the loss portion of the both the LDD and the excess policies is at risk of being much higher than expected. The loss portion is a much smaller percent of LDD premium as it is of excess premium, since LDD policies have more expenses. This means that the excess policy premium is at greater risk of being inadequate since there are fewer expenses to smooth it out. Also, under LDD policies, the insurer handles all claims, so it has more control over the ultimate loss amount. Excess policies may have uncertain ultimate excess losses because they are partially dependent on the TPA handling the claim below the retention.

EXAMINER'S REPORT

Candidates were expected to be able to calculate the premium for large dollar deductible (LDD) policies based on the parameters given in the question. In addition, candidates were expected to know advantages of LDD and excess policies.

Part a

Candidates were expected to fully calculate the prices for the expiring and Option 1 policies, based on the parameters given, and state the difference in price between the two policies.

Common mistakes include:

- Calculating the price for Option 2 (excess policy) instead of the expiring policy and calculating the difference between Option 1 and Option 2. These answers still received a majority of partial credit, as Option 1 being calculated correctly showed that the candidate understood the formula and how to use the parameters.
- Calculating incorrect entry ratios or looking up incorrect insurance changes from Modified Table M. Both of these were often due to using the aggregate deductible incorrectly to calculate the entry ratio or look up the insurance charge.
- Calculating incorrect excess and primary losses for either policy.
- Forgetting to multiply the insurance charge by expected **primary** losses and instead

multiplying by expected total losses.
- Incorrectly applying the expense parameters of the formula. Examples included forgetting to include $15K in fixed expenses, multiplying losses by one plus LBA and ULAE, or forgetting to include LBA or ULAE in the formula.
- Forgetting to calculate the difference between the two prices.

Part b

Candidates were expected to provide 2 advantages of the option which they selected (either Option 1 or Option 2) and provide full reasoning for each advantage. Alternatively, candidates could have provided an advantage of the option which they selected and a disadvantage of the option which they didn't select.

Common mistakes include:
- Citing advantages and disadvantages of a given option, but not connecting these to the situation presented (increasing ground up loss trend).
- Stating that the aggregate deductible provided a cap on losses to the insurer, rather than to the insured.
- Answering the question from the perspective of the insured, rather than the insurer.

QUESTION: 17	
TOTAL POINT VALUE: 2.5	**LEARNING OBJECTIVE(S): B2**
SAMPLE ANSWERS	
Part a: 1.5 points	
$R(250{,}000) = R_{data}(100{,}000)R_{curve}([250{,}000\text{-}100{,}000]/\text{average truncated \& shifted loss})$ Average loss = 38,000x0.62 + 100,000x0.23 + 200,000x0.07 + 300,000x0.5 + 500,000x.02 + 1,000,000x.01 = 95,560 → total losses = 9.556M $R_{data}(100)$ = 1- Losses under 100,000/total losses = 1- (62x38,000 + 38 x100,000)/9.556M = 0.3558 Avg. truncated & shifted losses = (100x7 + 200x5 + 400x2 + 900x1)/(7+5+2+1) = 3.4M/15 = 226,667 $R_{curve}([250{,}000 - 100{,}000]/226{,}667) = R_{curve}(0.662)$ Interpolate → (.662 - .6)/0.1 = 0.62 → 0.62x0.53 + (1-0.62)x0.58 = 0.549 R(250,000) = 0.3558x 0.549 = 0.1953	
Part b: 1 point	
<u>*Sample Responses for Methodology 1-Truncation*</u> • The excess premium will be more volatile through increasing the truncation point to 300k since there is very little data at that point. • The curve would have less data to fit to → more uncertainty → higher variance → higher xs ratios → higher xs WC premium. • If T went to $300k we would be relying more heavily on empirical data @ higher layers – data here is fairly thin so would be understating xs loss potential and xs ratios and premium. • Selecting a truncation point of 300,000 would mean the excess ratio for a limit of 250k would not use a fitted curve, R(250) would now be = (50x5+250x2+750)/9556 = 0.157. Therefore it would decrease the excess premium. <u>*Sample Responses for Methodology 2-Pareto*</u> • Increase the excess premium. Since Pareto has a heavier tail than exponential.	
EXAMINER'S REPORT	
Candidates were expected to calculate an excess ratio and comment on excess premiums given an aggregate loss distribution. Neither part comes directly from a specific page in the syllabus. Part a material is presented in the exhibits (including footnotes) while Part b material is spread throughout the reading. Candidates generally performed well on this question.	
Part a	
Candidates were expected to calculate an excess ratio from information provided. Common mistakes include: • incorrectly truncating losses at $250K • incorrectly interpolating • selecting the appropriate entry ratio (both approaches were given credit if correct)	

Part b
Candidates were expected to comment on the impacts on excess premiums given alternative methodologies. All reasonably argued conclusions were accepted. A common mistake was not directly assessing the impact to excess premium. Even when candidates could correctly describe the alternative methodologies, they did not state that this led to an increase in excess premium.

QUESTION: 18	
TOTAL POINT VALUE: 2	**LEARNING OBJECTIVE(S): C1,C2**
SAMPLE ANSWERS	
Part a: 1 point	

- Hazard module: limited scientific knowledge on earthquake thus leading to inaccurate frequency modeling
  Inventory module: outdated property information with inaccurate content information.
- Epistemic = CAT version of parameter risk
  Ex. Inventory module – not enough data on building codes
  Ex. Vulnerability module – not knowing impact on different types of buildings because they haven't been tested
- Epistemic – Parameter risk (reduce with more data)
  Inventory – Inaccurate/not enough data on building & their built value, type, etc.
  Loss – Lack of data on policy limits, deductibles, etc. to calculate insured loss.

Part b: 1 point

- Hazard Module - Pure randomness in the magnitude of the modeled earthquake.
  Vulnerability Module – Pure randomness in the susceptibility of a particular building to a given CAT event
- Aleatory = CAT version of process risk = random variation
  Hazard Module – The magnitude of earthquake that occurs is different than what is expected in the model
  Loss Module- The losses on homes after earthquake were higher than expected.
- Aleatory – process risk
  Vulnerability Module – the same exact earthquake could level a building or only damage part of it purely because of randomness of how strong shocks will be.
  Hazard Module – despite our best efforts to predict paths of hurricanes, there is inherent randomness in where a storm will hit
- Aleatory risk- nothing can be done, even if collect more data, it will not help us
  Loss module – exposures may change depending on time of day, if say earthquake occurs during work day under workers compensation policy.
  Vulnerability Module – even if we have an accurate prediction of the hazard and inventory, not all the situations can be taken into account, might miss something.

EXAMINER'S REPORT

Candidates were expected to demonstrate knowledge of the meanings of epistemic risk and aleatory risk, and give examples of these 2 types of risks in the catastrophe modules.

EXAM 8 FALL 2016 SAMPLE ANSWERS AND EXAMINER'S REPORT

Part a
Candidates were expected to correctly describe 2 of the 4 modules of CAT modeling (event/hazard, inventory/exposure, vulnerability/damage, loss), and to give an example of epistemic risk (parameter risk/lack of knowledge or data) that could be encountered in each of those 2 modules described. Candidates who described the modules correctly but were unclear about how they tied to epistemic risk received partial credit (example: "hazard module simulates catastrophes, which we can't predict well" – demonstrates knowledge of hazard module, but "can't predict well" could either be epistemic or aleatory depending on the reason, so this is an unclear example). Common answers that received partial credit included vague wording like "risk" or "uncertainty" without further explanation, since these could be interpreted as either aleatory risk or epistemic risk. To receive full credit the candidate was expected to demonstrate that epistemic risk is due to lack of data/understanding and/or that it can be reduced by additional data or a better model/parameters. Common mistakes include: • Describing epistemic risk as risk due to randomness (which is aleatory, not epistemic). • Giving a module description that didn't line up with the module name (for example, describing testing buildings for vulnerability as the "loss" module instead of "vulnerability" module). • Listing only the names of the modules (hazard, etc.) with no subsequent example or description (or an unclear/incorrect description). This did not receive credit because this did not demonstrate knowledge of the purpose of the modules or their risks.
Part b
Candidates were expected to correctly describe 2 of the 4 modules of CAT modeling (event/hazard, inventory/exposure, vulnerability/damage, loss), and to give an example of aleatory risk (process variance/inherent randomness) that could be encountered in each of those 2 modules described. Similar to part a, common answers that received partial credit included vague wording like "risk" or "uncertainty" without further explanation. To receive full credit the candidate was expected to demonstrate that aleatory risk is due to randomness and/or that it cannot be reduced by additional data or a better model/parameters. Common errors by candidates were: • Describing aleatory risk as risk due to lack of data/knowledge or issues with modeling/parameters (which is epistemic, not aleatory). • Giving a module description that didn't line up with the module name. • Listing only the names of the modules (hazard, etc.) with no subsequent example or description (or an unclear/incorrect description), similar to Part a.

QUESTION: 19	
TOTAL POINT VALUE: 3	**LEARNING OBJECTIVE(S): C1,C3**
SAMPLE ANSWERS	

Sample 1
Under XOL:
Ceded Loss:
Claim 1 = 2.5M
Claim 2 = 5M
Total = 7.5M

Ceded ALAE:
Claim 1 = (2.5/7.5)*0.5 = 0.5M
Claim 2 = 0.25M
Total = 0.75M

1% AEP w/ XOL - Company must hold $70M based on curve.
Under QS, capital held is $100M – ($50M)*(0.2)*(1.5) = $85M

Ceded Premium under XOL = 0.18*50M = 9M
Ceded Premium under QS = 0.2*(50M)*(1-0.3) = 7M

Retained Premium (net of commission) under XOL ($50M - $9M) = $41M
Expenses = $15M
Retained Loss & ALAE = $30M - $7.5M – 0.75M = $21.75M
Profit = $41M - $15M - $21.75M = $4.25M
ROE = $4.25M/$70M = 6.07%
Impact on ROE = 6.07 – 5 = 1.07%

Retained Premium (net of commission) under QS ($50M - $7M) = $43M
Expenses = $15M
Retained Loss & ALAE = $30M*(.8) = $24M
Profit = $43M - $15M - $24M = $4M
ROE = $4M/$85M = 4.71%
Impact on ROE = 4.71 – 5 = -0.3%

Sample 2 (just for the QS part)
Profit = $50M*(0.8) - $15M - $30M*(0.8) + 0.3*($50M)*(0.2) = $4M
$15M max cession
100 year MPL is 130 @ 1%
Capital held = 130-115 = 115
ROE = $4M/$115M = 3.47%
Impact on ROE = 3.47% - 5% = 1.53%

Note: Credit was also given if there was explanation regarding how the max LR affected the capital requirement without having the calculation itself.
Sample 3

XOL
Claim 1 = 7.5m-5m=2.5m loss
(2.5m/7.5m)(1.5m)=500k ALAE
3m total
Claim 2=10m-5m=5m
(5m/10m)(500k)-250k
5.25m total

Total loss and ALAE=30m-3m-5.25m=21.75m
Premium=50m(1-0.18)-41m
41m-21.75m-30m-15m=4.25m
Versus 50m-30m-15m=5m

However the company only has to hold 70m instead of 100m so the extra 30m can be invested to make up for the 750k difference which will result in higher return on capital

Quota share
Total loss and alae=30m(.8)=24m
Premium=50m(.8)+50m(.2)(.3)=43m

43m-24m-15m=4m
Ceded premium=50m(.2)=10m
10m(1.5)=15m

This means a 100m loss results in an 85m net loss, this is only a difference of 15m as opposed to 20m which is the quota share of the capital and therefore reduces the return on capital

EXAMINER'S REPORT

Candidates were expected to understand types of reinsurance contracts and common provisions in reinsurance contracts.

Many candidates did not know or understand the capital requirements. Credit was given if explanation was given on how they interpreted the graph.

Full credit was given if the provisions for the reinsurance contracts were calculated correctly, the capital requirements determined and the ROEs calculated or if explanations were given rather than the ROE calculations themselves.

Common mistakes include:

- Not using the correct ratio to determine the pro-rata ALAE.
- Not calculating the commission appropriately (some used the % of losses or the % of entire premium rather than ceded premium).
- For the QS treaty, using the individual claims rather than the total losses.
- For the QS treaty, not understanding the impact of the max loss ratio on the capital requirement.
- For the XOL treaty, mixing up what was retained vs. what was ceded.

QUESTION: 20

TOTAL POINT VALUE: 3 | **LEARNING OBJECTIVE(S): C3**

SAMPLE ANSWERS

Part a: 0.75 point

Sample 1

Loss	A 100x200	B 100x300	C 100x400	Primary insurer	Total
1	100	80	0	200	380
2	60	0	0	200	260
3	40*	100	100	360	600

*since only one reinstatement, assuming there is no coverage after layer is filled twice

Sample 2

Reins	Layers	Loss amount 380	260	600	(capped at 0)
A	100xs200	100	60	100	=Min[100;L-200]*100%
B	100xs300	80	-	100	=Min[100;L-300]*100%
C	100xs400	-	-	100	=Min[100;L-400]*100%
	Total ceded	180	60	300	

The primary insurer retains (380-180)=$200 for the first event, (260-60)=$200 for the second event and (600-300)=$300 for the third event

*I assumed that more than one reinstatement was possible but at least one was mandatory

Sample 3

Assume reinsurer A treaty inures to the benefit of reinsurer B which inures to the benefit of reinsurer C treaty

Loss	Ceded to contract A	Ceded to contract B	Ceded to contract C	Retained
1	100	0	0	280
2	60	0	0	200
3	40	100	60	400

Part b: 2.25 points

Sample 1

	A	B	C
Reins premium	= 10%(100)=$10	.07(100)=$7	.04(100) = $4
Loss			
1	100/100($10)=$10	80/100($7)=5.6	0
2	0	0	0
3	0	20/100($7)=1.4	100/100($4)=4

|->(can only pay one reinstatement)

A=$10 B=$7 C=$4

Sample 2

Rate on line = limit/premium
Reinstatement premium = premium x 100% x loss in layer/limit

Reinsurer A
Premium = 100*0.1 = 10

- Event 1 reinstatement premium: 10$ x 100% x 100/100 = 10$
- Event 2 reinstatement premium: 10$ x 100% x 60/100 = 60$
- Event 3 reinstatement premium: 10$ x 100% x 100/100 = 10$

Reinsurer B
Premium = 100*0.07 = 7

- Event 1 reins premium: 7 x 100% x 80/100 = 5.6$
- Event 3 reins premium: 7 x 100% x 100/100 = 7$

Reinsurer C
Premium = 100*0.04 = 4

- Event 3 reins premium: 4 x 100% x 100/100 = 4$

*I assumed that more than one reinstatement was possible but at least one was mandatory

Sample 3
Assume reinsurer A treaty inures to the benefit of reinsurer B which inures to the benefit of reinsurer C treaty

		[1]	[2]	[1]*[2]
Initial premium:	Reinsurer	Limit	Rate on line	Premium
	A	100	10%	10
	B	100	7%	7
	C	100	4%	4

Loss event	Ceded loss A	ReinstatementA	CLB	ReinB	CLC	ReinC
1	100	10=(100/100)(10)	0	0	0	0
2	60	0	0	0	0	0
3	40	0	100	(100/100)(7)=7	60	(60/100)(4)=2.4

EXAMINER'S REPORT

Candidates were expected to demonstrate that they understood how to calculate ceded and retained losses under a catastrophe reinsurance treaty, and also how to calculate the reinstatement premiums owed to the reinsurers after loss events.

In this case each treaty was allowed a single reinstatement and each reinsurer could pay up to double the treaty limit. However, many candidates interpreted the term "single mandatory reinstatement" to mean that the primary reinsurer MUST reinstate the coverage at least once but could possibly reinstate more than one time. When candidates stated their assumption, full credit was given for this alternate assumption and partial credit was given when the assumption was not stated.

Part a

Candidates were expected to be able to calculate how much was retained by the primary insurer given that each reinsurance treaty had a single reinstatement and that there was no more coverage after treaty limit had been fully exhausted twice.

Many candidates assumed that multiple reinstatements were possible and that only the first reinstatement was mandatory. If those candidates stated their assumption regarding the reinstatements, full credit was given as long as the calculations were executed correctly according to that assumption. When the assumption was not explicitly stated, partial credit was given.

Common mistakes include:

- Not showing any work to explain how the amount retained was calculated.
- Not stating any assumptions.
- Not realizing that reinsurer A still had to pay $40 on loss event 3 seeing as the treaty limit had not been fully exhausted on loss event 2.

Part b

Candidates were expected to be able to calculate the amount ceded to each reinsurer and the reinstatement premium that was paid to each reinsurer after each loss event. Candidates were expected to demonstrate that they understood the reinstatement premium is calculated as the (Ceded Loss / Limit) * Initial Treaty Premium.

Common mistakes include:

- Not showing enough work to demonstrate how any element of the reinstatement premium was calculated.
- Not stating any assumptions.
- Calculating the initial treaty premium wrong or not showing details of calculations.
- Not being clear enough about which event or which reinsurer the reinstatement premiums were for.
- Not realizing that reinsurer B would receive a second reinstatement premium on loss event 3 as they had not fully exhausted their limit after loss event 1 and therefore only a partial reinstatement premium was paid after this loss event.

QUESTION: 21	
TOTAL POINT VALUE: 4	**LEARNING OBJECTIVE(S): C3,C5**

SAMPLE ANSWER

<u>IV range</u> <u>Avg in Range</u>
1-4M 2.5M

$\frac{E[x;1M]}{E[x]} = \frac{10 * 1M + 22M}{40M} = 0.8$ which corresponds to $\frac{1M}{2.5M} = 0.4$ of IV

This matches Y_4 curve better, since 0.8 is closer to 0.82 (40% IV of Y_4) vs. 0.72 (40% IV of Y_3).

XS treaty: 4M xs 4M

Need exposure factor for policies with IV between 4M and 8M (avg IV = 6M) → G (8/6) – G(4/6)

For Y_4, c=4

$b(4) = e^{3.1 - 0.15(4)(1+4)} = 1.11$

$g(4) = e^{4(0.78+0.12*4)} = 154.47$

G(8/6) = G(133.3%) = 1.0 (use 1.0 for IV ≥100%)

$G(4/6) = G(66.7\%) = \frac{\ln(((154.47 - 1)*1.11 + (1 - 154.47*1.11)*1.11^{.667})/(1 - 154.47))}{\ln(154.47 * 1.11)} = 92\%$

Ground-up Loss Ratio = 40M/100M = 0.4

Losses covered by treaty = (1 – 0.92) * 5M * 0.4 = 163,148

Ceded premium = 0.01 * (25M + 5M) = 300K

Ceded Loss Ratio = 163,148/300,000 = 54.4%

EXAMINER'S REPORT

Candidates were expected to synthesize material from both Bernegger and Clark, making this question more complex than seen on prior exams. Candidates were expected to complete the following steps:

- Selection of the best-fitting exposure curve
- Computation of the proportion of losses falling in the treaty
- Calculation of the ceded loss ratio

To select the best fitting exposure curve, candidates were expected to recognize that the historical experience was limited to Insured Values (IVs) of \$1M-\$4M and that the experience losses provided could be used to find the best fitting exposure curve.

Candidates who took the steps to calculate the % of Cumulative Loss under \$1M (80%) and Average Percent of IV at \$1M (40%) generally demonstrated the ability to choose a curve using the table provided.

Common mistakes when selecting the curve include:

- Not noticing that the loss provided for claims exceeding \$1M was ground-up loss. To figure out what percent of cumulative loss was under \$1M, candidates needed to recognize that 10 claims that were >\$1M had \$1M of loss each that is under \$1M that was not accounted for in the \$22M associated with the claims under \$1M. So the total loss less than \$1M was \$32M = \$22M + 10 claims * \$1M / claim.
- Not recognizing that historical IVs were not all \$4M but instead were spread between \$1M and \$4M was important in selecting the % of Insured Value to look up in the table provided for Y3 and Y4. Examples in Clark suggest that we can assume an even distribution of insured values and select the midpoint Insured Value (\$2.5M) to choose the ratio to use to look up factors in a table of this sort.

A common misunderstanding in computing the proportion of losses excess of the treaty attachment point was confusing per-risk excess treaty pricing with layer pricing where the limit of loss is above the layer. To get full credit, candidates needed to know:

- Values of c for the various Swiss Re curves. For Y3, c=3 and for Y4, c=4.
- G(x) for the MBBEFD curve represents the % of cumulative loss less than x, where x is a % of IV (or Max Probable Loss for unlimited distributions)
- With a range of IVs the average IV should be used (see Clark). Using G(8/8)-G(4/8) for the \$4M to \$8M exposure factor overstates losses in range.

Common mistakes in the calculation of the final ceded loss ratio include:

- Not recognizing that the treaty premium in the denominator of the loss ratio is the full subject premium of \$30M times the 0.01 treaty rate.
- Not realizing the subject premium associated with the IV range <\$4M does not generate losses exposing the excess treaty.
- Not recognizing that the expected loss ratio for IVs \$4-8M should be assumed to be the same as that of \$1-4M (see examples in Clark).