Exam 8

CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

Exam 8

Advanced Ratemaking

October 28, 2015

4 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 59.5 point examination consists of 23 problem and essay questions.

2. For the problem and essay questions, the number of points for each full question and part of a question is indicated at the beginning of the question or part. Answer these questions on the lined sheets provided in your Examination Envelope. Use dark pencil or ink. Do not use multiple colors or correction fluid/tape.

   - Write your Candidate ID number and the examination number, 8, at the top of each answer sheet. Do not use leading zeroes. Your name, or any other identifying mark, must not appear.
   - Do not answer more than one question on a single sheet of paper. Write only on the front lined side of the paper – **DO NOT WRITE ON THE BACK OF THE PAPER**. Be careful to give the number of the question you are answering on each sheet. If your response cannot be confined to one page, please use additional sheets of paper as necessary. Clearly mark the question number on each page of the response in addition to using a label such as "Page 1 of 2" on the first sheet of paper and then "Page 2 of 2" on the second sheet of paper.
   - The answer should be concise and confined to the question as posed. When a specific number of items is requested, do not offer more items than the number requested. For example, if three items are requested, only the first three responses will be graded.
   - In order to receive full credit or to maximize partial credit on mathematical and computational questions, you must clearly outline your approach in either verbal or mathematical form, showing calculations where necessary. Also, you must clearly specify any additional assumptions you have made to answer the question.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

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4. Prior to the start of the exam you will have a **fifteen-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. A chart indicating the point value for each question is attached to the back of the examination. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.

   - Verify that you have received the reference materials:

   a. National Council on Compensation Insurance, Experience Rating Plan Manual for Workers Compensation and Employers Liability Insurance (*Excerpt from 2015 Study Kit*).
   b. Insurance Services Office, Inc., Commercial General Liability Experience and Schedule Rating Plan.
   c. National Council on Compensation Insurance, Retrospective Rating Plan Manual for Workers Compensation and Employers Liability Insurance (*Excerpt from 2015 Study Kit*).

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until two hours after the start of the examination. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.

7. At the end of the examination, place all answer sheets in the Examination Envelope. Please insert your answer sheets in your envelope in question number order. Insert a numbered page for each question, even if you have not attempted to answer that question. Nothing written in the examination booklet will be graded. Only the answer sheets will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope. Interoffice mail is not acceptable.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by November 14, 2015.

END OF INSTRUCTIONS

1. (2.5 points)

An actuary is evaluating a merit rating plan for private passenger cars. Given the following:

Number of Accident-Free Years	Earned Car Years	Number of Claims Incurred
2 or More	500,000	20,000
1	200,000	15,000
0	100,000	9,000
Total	800,000	44,000

- Frequency varies by territory.
- State law prohibits reflecting territory differences in rating.
- Annual claims for an individual driver follow a Poisson distribution.
- Claim cost distributions are similar across all drivers.

a. (0.5 point)

Identify one potential issue with the exposure base used. Briefly explain whether or not earned premium would be a better choice for the exposure base.

b. (1.0 point)

Calculate the credibility of one driver with one or more year's accident-free experience.

c. (1.0 point)

Calculate the credibility of one driver with 0 Accident-Free years.

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2. (2.75 points)

An actuary is modeling claim frequency for a portfolio with the following distribution of exposures.

	Vehicle Class			
Territory	Car	Van	Truck	Other
A	10,000	2,000	0	0
B	2,000	5,000	0	0
C	0	0	5,000	0
D	30	0	0	3,000

The actuary proposes a generalized linear model (GLM) with the following parameterization.

Territory	
Factor level	Parameter
A	β_1
B	β_2
C	β_3
D	β_4

Vehicle Class	
Factor level	Parameter
Car	β_5
Van	β_6
Truck	β_7
Other	β_8

a. (1.0 points)

Briefly discuss how intrinsic and extrinsic aliasing are present in this analysis using examples from the data. For each type of aliasing briefly explain the potential impact on the results.

b. (0.5 point)

Provide one example of near aliasing in this analysis and briefly describe any potential impact on the modeling results.

c. (1.25 points)

Propose an alternative GLM approach to avoid extrinsic, intrinsic, and near aliasing. Describe how many covariates would be required.

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3. (2.5 points)

An actuary is considering using a generalized linear model to estimate the expected frequency of a recently introduced insurance product.

Given the following assumptions:

- The expected frequency for a risk is assumed to vary by state and gender.
- A log link function is used.
- A Poisson error structure is used.
- The likelihood function of a Poisson is

  $$l(y;\mu) = \sum \ln f(y_i;\mu_i) = \sum -\mu_i + y_i \ln \mu_i - \ln(y_i!)$$

- β_1 is the effect of gender = Male.
- β_2 is the effect of gender = Female.
- β_3 is the effect of State = State A.

Claim Frequency

	State A	**State B**
Male	0.0920	0.0267
Female	0.1500	0.0500

Given that $\beta_3 = 1.149$, determine the expected frequency of a male risk in State A.

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4. (2.25 points)

An actuary is reviewing an account that has been with the company for over ten years.

Given the following:

- The claim frequency for this account follows a Poisson distribution, with $\lambda = 0.012$
- The recorded frequency for the last five years is as follows:

Year	Exposures	Frequency
2010	9,500	0.011
2011	11,000	0.010
2012	13,000	0.013
2013	10,500	0.012
2014	12,000	0.010

- The critical value for the relevant Chi-squared distribution is 9.49

a. (1.5 points)

Use the Chi-squared test to evaluate whether the claim frequency is shifting over time.

Include the hypotheses, test statistic, and provide an interpretation of the result.

b. (0.75 points)

Fully describe another method for determining whether claim frequency is shifting over time.

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5. (2.5 points)

An actuary estimated the loss cost for workers compensation insurance using a multi-dimensional credibility method.

Given the following:

- There were 2 classes in Hazard Group X.
- There were no major or minor permanent partial losses.
- Premium information was not available.
- Holdout sample of odd years was used as a proxy of the true mean.

Claim Count by Injury Type for Hazard Group X

	Even Year 1			Even Year 2		
Class	Fatal (F)	Permanent Total (PT)	Temporary Total (TT)	Fatal (F)	Permanent Total (PT)	Temporary Total (TT)
Class 1	2	10	1,000	1	12	1,000
Class 2	3	10	1,000	2	13	1,000
Total	5	20	2,000	3	25	2,000

Optimal Weights for Estimation of Permanent Total Injury Ratio

Fatal	Permanent Total
0.2	0.3

a. (1 point)

Determine the ratio of permanent total injury to temporary total injury for Class 2 using a multi-dimensional credibility method.

b. (1 point)

Fully describe the steps involved in performing a quintile test to evaluate the actuary's work.

c. (0.5 point)

Briefly describe one shortcoming of the individual class sum of squared errors test and briefly describe why the quintiles test is a better way to evaluate the actuary's work.

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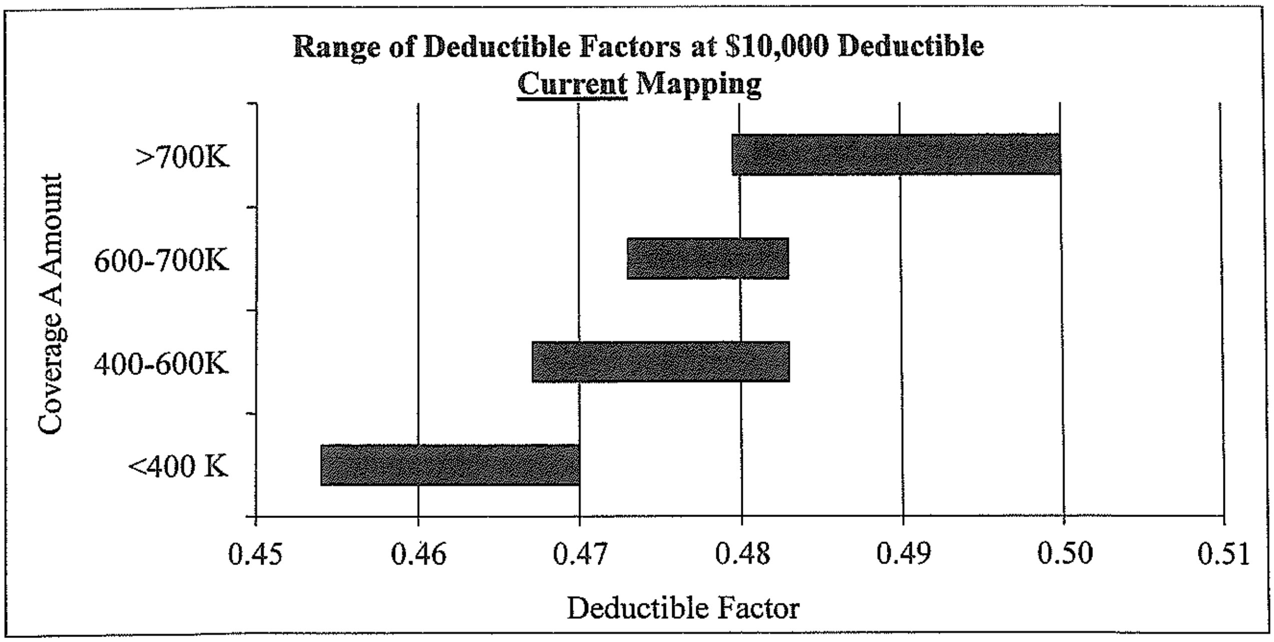
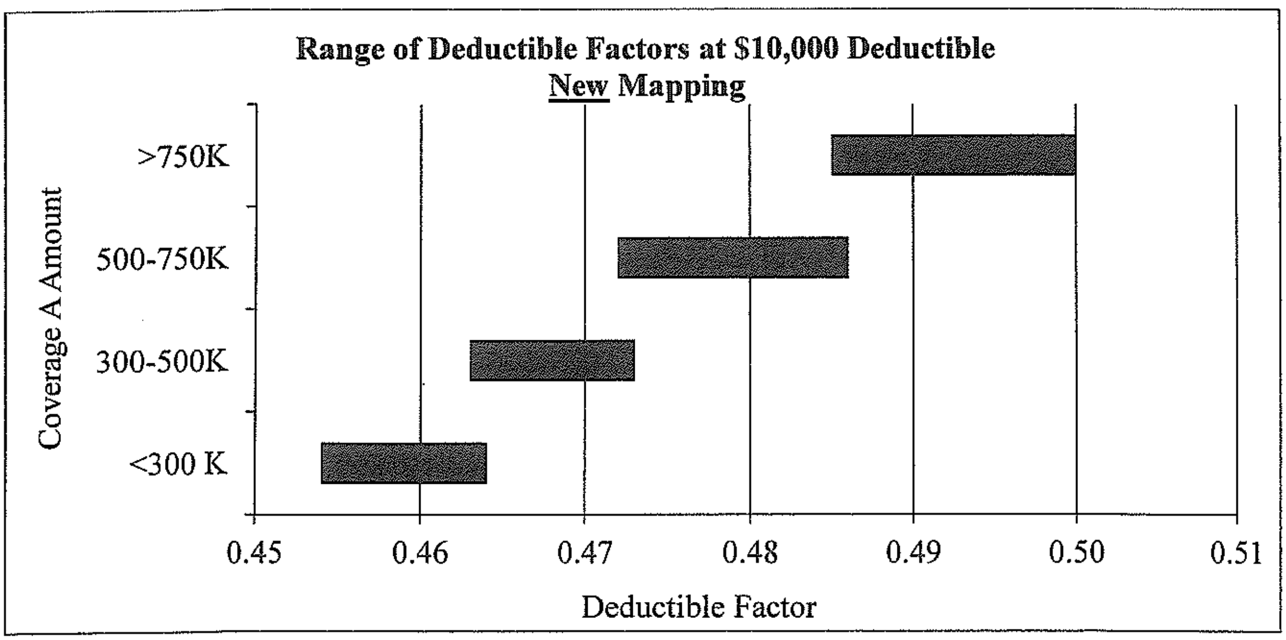
6. (3 points)

A company groups its homeowners' policies based on Coverage A amount for ratemaking. The company is proposing using a new method, k-means clustering, to group these policies. The graphs below show the range of deductible factors by Coverage A amount group for the current and proposed method:





<<QUESTION 6 CONTINUED ON NEXT PAGE>>

a. (1 point)

Describe the steps in performing the k-means clustering method.

b. (0.5 point)

Discuss whether the current or proposed method should be used to group homeowners' policies using the two graphs provided.

c. (1.5 point)

Identify two operational considerations that would affect the decision to implement a change in policy grouping and explain how these considerations would apply to implementation of the new groups.

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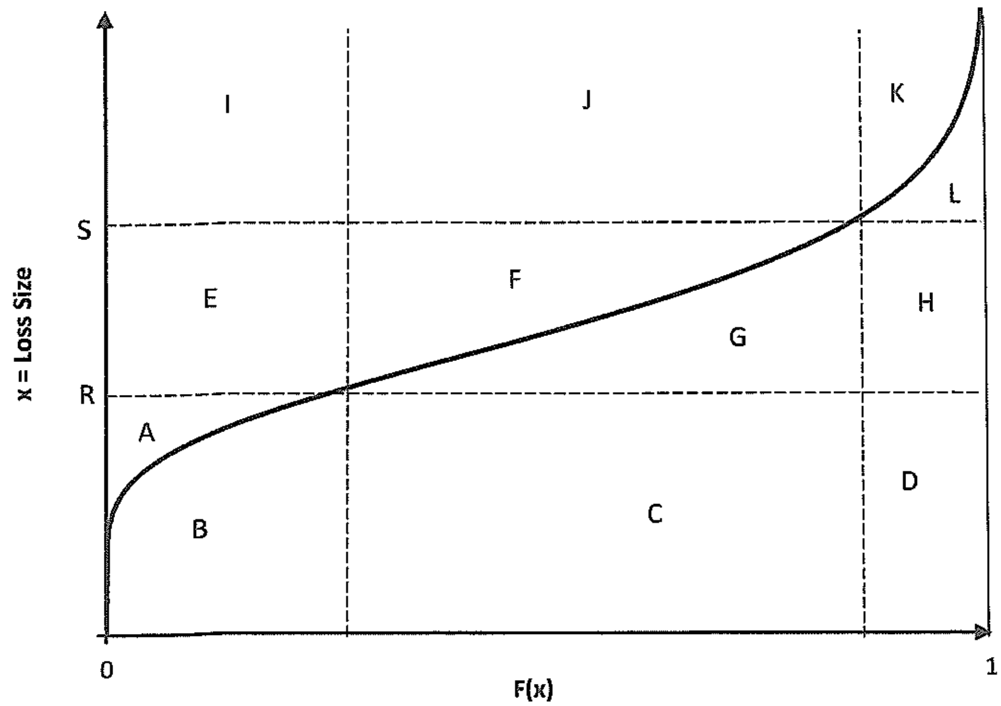
7. (2 points)

The following Lee diagram applies to a cumulative size of loss distribution F(x), where letters A through L represent the areas of the enclosed regions.



a. (0.25 point)

Express the area G + H in integral form, using the layer method.

b. (0.25 point)

Express the area B + C + D in integral form, using the size method.

c. (0.25 point)

Assume R is the basic limit. Express the increased limits factor for limit S algebraically using the area labels provided in the graph.

d. (1.25 points)

Describe the consistency test for increased limit factors. Use a graph to explain what the consistency test is evaluating. Label all relevant features of the graph.

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8. (2.5 points)

An actuary is working with ground-up historical loss data and is considering fitting one continuous curve to this data to calculate ILFs for higher limits.

The last time such an analysis was conducted, empirical losses were used to determine ILFs directly without fitting a continuous curve to the data.

a. (1 point)

Provide two shortcomings of using empirical data to determine ILFs and briefly describe how curve fitting may overcome each of these shortcomings.

b. (1.5 points)

There is a concern that fitting one continuous curve to the entire distribution of losses will overstate losses over certain intervals and understate losses over other intervals. Propose and fully describe a solution that addresses this concern while still incorporating an element of curve fitting in the solution.

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9. (3.25 points)

Given the following Premises/Operations General Liability loss experience evaluated as of September 1, 2013:

Policy Effective Date	Policy Type	Total Ground-Up Incurred Loss	Total Ground-Up Incurred ALAE
March 1, 2010 to February 28, 2011	Occurrence	1,500,000	600,000
March 1, 2011 to February 29, 2012	Occurrence	400,000	400,000
March 1, 2012 to February 28, 2013	Occurrence	350,000	2,000,000
March 1, 2013 to February 28, 2014	Occurrence	150,000	20,000

- The insured has experienced the following ground-up large losses:

Accident Date	Incurred Loss	Incurred ALAE
June 30, 2010	700,000	500,000
December 31, 2011	150,000	200,000
April 5, 2012	55,000	60,000

- Annual Basic Limits Premium = $800,000.
- Expected Loss and ALAE Ratio = 80%.

A new policy will become effective March 1, 2014 to February 28, 2015 and will be written on an occurrence basis.

Using the ISO Commercial General Liability Experience and Schedule Rating Plan, calculate the experience modification factor used to price this policy.

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10. (3 points)

One common expression for the experience modification for a single-split plan is:

$$M = 1 + Z_p \frac{A_p - E_p}{E} + Z_e \frac{A_e - E_e}{E}$$

where:

- M is the modification factor
- Z_p and Z_e are credibility constants
- A_p is the actual primary loss
- A_e is the actual excess loss
- E_p is the expected primary loss
- E_e is the expected excess loss
- E is the expected total loss.

a. (0.75 point)

In the right-hand side of the equation above, there are three terms separated by '+' signs. Briefly describe the role that each term serves in computing the experience mod.

b. (0.5 point)

Of the two credibility constants, Z_p and Z_e, identify which of the two is typically the larger in magnitude, and explain why.

c. (1.75 points)

Determine the effectiveness of each of the following credibility functions and select which function is the most appropriate.

	Credibility			
Expected Loss	Function 1	Function 2	Function 3	Function 4
1,000	15%	65%	55%	80%
2,000	35%	75%	63%	75%
3,000	55%	85%	70%	62%
4,000	75%	95%	76%	53%
5,000	95%	105%	81%	40%

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11. (2.5 points)

An underwriter and an actuary are discussing the effectiveness of the current experience rating plan. The following table contains experience from five experience rated risks (all of similar size):

Risk	Manual Premium	Modified Premium	Actual Losses
1	400,000	360,000	300,000
2	600,000	840,000	690,000
3	800,000	560,000	440,000
4	900,000	1,080,000	860,000
5	1,000,000	800,000	650,000

a. (1.5 points)

Evaluate whether the experience rating plan is effective or not and explain why.

b. (1 point)

The underwriter argues that the modification factor for risk 4 is too high. Propose two additional pieces of information the actuary could request regarding risk 4 in order to support or disprove the underwriter's argument and explain why the information would be useful.

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12. (1.5 points)

An actuary is evaluating the effectiveness of an experience rating plan and has calculated the following values:

	Standard Loss Ratio		Sample Variance in Loss Ratios	
Risk size	Risks with credit mod	Risks with debit mod	Unmodified	Modified
Small	1.05	1.08	0.07	0.008
Medium	0.98	0.96	0.05	0.004
Large	0.99	1.00	0.04	0.004

a. (0.5 point)

Evaluate whether this plan satisfies the necessary condition for proper credibility.

b. (0.5 point)

Determine which risk size has the most accurate experience rating based on the efficiency test.

c. (0.5 point)

It has been determined that premiums are inadequate for small risks. Discuss whether premium inadequacy is better corrected by changing the manual rates or the experience rating plan.

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13. (3.25 points)

An actuary prices two loss-sensitive options for a workers compensation policy as follows:

Option 1: A large deductible plan with a per-occurrence deductible of $50,000

Option 2: An incurred retrospective rating plan with the following parameters:

Per Occurrence Limit	$50,000
Basic Premium	$150,000
Tax Multiplier	1.045
Loss Conversion Factor	1.100
Deposit Premium (paid at policy inception)	$1,000,000

For each of the options above, assume that no aggregate limits or maximum premiums apply and that the first adjustment will take place 18 months after policy inception.

Additionally, the actuary has developed the following assumptions for the insured:

	Unlimited	Limited to $50,000
Expected Loss	$650,000	$435,000
18-Ultimate Incurred LDF	4.25	3.75
18-Ultimate Paid LDF	8.80	6.55

a. (2.25 points)

For each of the plans above, determine the expected cash flows between the insured and insurer 18 months after policy inception.

b. (1 point)

The insured is contemplating a third option of purchasing an excess policy with a self-insured retention of $50,000.

i. Which of the three options would be least attractive to the insurer if they wish to minimize credit risk? Briefly explain your choice.

ii. Which of the three options would be least attractive to the insurer if they wish to minimize interest rate risk? Briefly explain your choice.

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14. (4 points)

An insured has a large dollar deductible (LDD) policy. Total losses and ALAE limited to the deductible are distributed uniformly on the interval [0, 400,000], and total unlimited losses and ALAE are distributed uniformly on the interval [0, 800,000].

- The insured currently has an aggregate loss limit of $300,000.
- Credit risk is not contemplated in pricing.
- The deductible applies to both loss and ALAE.

The following expenses apply to this insured:

Expense Item	**Value**	**Applies to**
ULAE	7.5%	Loss & ALAE
Loss Based Assessments	5%	Loss & ALAE
Overhead	$45,000	Fixed
Acquisition	6%	Written Premium
Commission	12.5%	Written Premium
Premium Tax	4%	Written Premium
Profit and Contingency	-5%	Written Premium

a. (2 points)

Calculate the LDD premium for this insured.

b. (2 points)

It is later determined that, although the distribution of total unlimited losses and ALAE remains unchanged, the total losses and ALAE limited to the deductible actually follow the following distribution:

- 75% probability of loss and ALAE between $0 and $300,000
- 25% probability of loss and ALAE between $300,000 and $700,000
- Losses follow a uniform distribution within each range.

Use one or more Lee diagrams to demonstrate the impact to the premium for the LDD policy.

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15. (2.5 points)

An insured in a retrospectively-rated workers compensation plan currently pays a basic premium of $26,820. The following parameters apply to the insured's policy:

Standard Premium	$100,000
Expense Ratio (e)	$20,000
Expected Losses	$70,000
Tax Multiplier	1.00
Loss Conversion Factor	1.17
Entry Ratio @ G	1.00
Entry Ratio @ H	0.75

The insured believes that the insurance charge embedded in the current basic premium is unfair and cites the following unlimited loss ratios from five similarly-sized competitors doing business in the same industry:

Competitor	**Loss Ratio**
1	70.0%
2	105.0%
3	52.5%
4	87.5%
5	35.0%

a. (2 points)

Compare the net insurance charge in the current basic premium for this policy to the net charge based on the provided competitor loss ratio experience.

b. (0.5 point)

Discuss the appropriateness of using the basic premium derived from the competitor data to price this policy.

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16. (2.5 points)

An actuary prices a retrospectively rated policy with effective date January 1, 2015 using the NCCI Retrospective Rating Plan Manual. The following information applies:

Maximum premium	\$12,300,000
Expected loss	\$4,000,000
Expense provision (excluding taxes)	\$1,629,440
Loss conversion factor	1.2
Tax Multiplier	1.025
State Hazard Group Differential	0.98

The plan has no specified minimum premium and no per occurrence limit.

a. (2 points)

Calculate the loss at maximum premium that balances the plan.

b. (0.5 point)

Calculate the basic premium.

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17. (2 points)

An actuary prices a retrospectively rated policy based on the assumption that aggregate losses follow a uniform distribution between \$0 and \$1,000,000. The actuary determines that the following provisions result in a balanced plan:

Standard premium	\$700,000
Loss at minimum premium	\$80,000
Loss at maximum premium	\$750,000
Basic premium	\$83,660
Loss conversion factor	1.2

Assume there are no taxes.

a. (1.5 points)

Calculate the implicit premium discount associated with the plan.

b. (0.5 point)

Briefly describe how premium discount is treated in a retrospectively rated policy compared to a guaranteed cost policy.

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18. (2 points)

An actuary is given the following claims experience for large dollar deductible workers compensation insurance for five identically sized risks. Each claim is a separate occurrence.

	Individual Claims Experience (Gross of Deductible)			
Risk #	**Claim 1**	**Claim 2**	**Claim 3**	**Claim 4**
1	70,000	80,000		
2	165,000	300,000	250,000	
3	150,000			
4	150,000	250,000	200,000	150,000
5	250,000			

Calculate the expected loss cost for a risk identical to the five above with a per-occurrence deductible of $150,000 and an annual aggregate deductible of $450,000.

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19. (1.5 points)

An actuary has priced a large dollar deductible policy using the information given below and has indicated a premium of $273,500. Assume no collateral is held on behalf of the insured.

Standard Premium	1,000,000
Ground-up Expected Loss and ALAE Ratio	65%
Excess Ratio (% of Total Loss and ALAE)	10%
ULAE Load (% of Total Loss and ALAE)	8%
General Expenses (% of Standard Premium)	5%
Credit Risk Load (% of Standard Premium)	**X%**
Acquisition Expenses (% of Net Premium)	5%
Taxes (% of Net Premium)	7%
Profit and Contingencies (% of Net Premium)	5%

a. (1 point)

Calculate X, the implied Credit Risk Load in the actuary's indicated premium.

b. (0.5 points)

Explain why profit loads can be higher for large dollar deductible policies compared to excess policies.

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20. (2.5 points)

An actuary is using the following exposure curve to rate a non-proportional reinsurance treaty:

$$G(x) = \frac{(1 - b^x)}{(1 - b)}$$

The actuary is also given the following information:

Maximum Possible Loss	\$5,000,000
Insured Value	\$5,000,000
Gross Premium	\$6,000
Expected Loss Ratio	60%
Retention of non-proportional reinsurance treaty	\$150,000
Expected Ceded Risk Premium	\$2,705

a. (0.5 point)

Briefly describe a method to allocate gross premium for the non-proportional reinsurance treaty between the ceding company and the reinsurer.

b. (0.5 point)

Given the probability of a total loss is 0.03, calculate the parameter b in the formula above.

c. (1.5 points)

Given the answer in part b. above, calculate the limit of the non-proportional reinsurance treaty.

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21. (3.75 points)

A reinsurer is offering a ceding company a two-year aggregate stop loss with the following terms:

- The treaty is effective on 1/1/2016 and expires on 12/31/2017.
- The treaty will cover aggregate losses between a 65% and a 70% loss ratio for accident years 2016 and 2017 separately.
- Premium is paid at the beginning of each year and all losses are paid at the end of the year incurred.

Additionally, the reinsurance treaty has the following termination provisions:

- The ceding company has the option to terminate the contract at the end of 2016 and receive a profit commission from the reinsurer calculated as follows:
  - Profit commission = [35% * 2016 Ceded Premium] – [2016 Ceded Loss], subject to a minimum of 0.
- If the contract is not terminated at the end of 2016, the ceding company will again have the option to terminate the contract at term expiration and receive a profit commission from the reinsurer calculated as follows:
  - Profit commission = [35% * Full Term Ceded Premium] – [Full Term Ceded Loss], subject to a minimum of 0.
- The reinsurer cannot terminate the contract at any time.

<<QUESTION 21 CONTINUED ON NEXT PAGE>>

The reinsurer has simulated five trials of the ceding company's loss ratio subject to the aggregate stop loss as follows (values in millions):

	Accident Year			
	2016		2017	
Simulation	Loss Ratio	Loss Amount	Loss Ratio	Loss Amount
1	73.9%	517	68.0%	510
2	64.8%	454	67.9%	509
3	66.8%	468	65.5%	491
4	65.6%	459	65.3%	490
5	65.1%	456	58.7%	440
Mean	67.2%	471	65.1%	488
Subject Premium	700		750	
Ceded Premium	9.8		10.5	

For each simulation, assume that the ceding company will only terminate the contract if the profit commission payable is greater than zero.

a. (1 point)

Calculate the expected profit commission payable at the end of 2016.

b. (2.75 points)

Calculate the expected profit commission for the full term of the contract.

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22. (2.75 points)

An insurance company is exposed to three independent catastrophic risks in three different regions in a given year. More than one event can occur in a year but each event can only occur once in a year. Events have the following size and probability:

Event	Loss Amount	Annual Probability of Occurrence
1	$10,000,000	0.10
2	$15,000,000	0.05
3	$35,000,000	0.02

a. (2.25 points)

Calculate the Aggregate Exceedance Probabilities associated with the insurance company's exposure.

b. (0.5 points)

Using a randomly generated number of 0.86, simulate the insured total loss.

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23. (2.5 points)

An actuary is pricing the $9,000,000 excess of $1,000,000 layer for an excess of loss policy. Total insured value of the properties is $10,000,000. Historical information for this policy is as follows:

Accident Year	Excess Loss Development Factor	On-Level Trended Premium
2012	1.01	$5,200,000
2013	1.05	$5,700,000
2014	1.10	$5,900,000

Accident Date	Actual Ground-Up Loss	Trend Factor	Trended Losses
July 18, 2012	$3,500,000	1.10	$3,850,000
February 12, 2013	$2,000,000	1.08	$2,160,000
August 15, 2013	$1,000,000	1.07	$1,070,000
March 3, 2014	$3,000,000	1.04	$3,120,000
November 2, 2014	$920,000	1.01	$929,200

The exposure curve below applies to the insured risk:

% of Insured Value	Exposure Factor
10%	30%
20%	41%
30%	52%
40%	63%
50%	71%
60%	75%
70%	80%
80%	84%
90%	89%
100%	90%
120%	100%

Calculate the policy's loss cost as a percentage of premium.

END OF EXAMINATION

Exam 8
Advanced Ratemaking

POINT VALUE OF QUESTIONS

QUESTION	VALUE OF QUESTON	SUB-PART OF QUESTION (a)	(b)	(c)	(d)	(e)	(f)	(g)
1	2.50	0.50	1.00	1.00				
2	2.75	1.00	0.50	1.25				
3	2.50							
4	2.25	1.50	0.75					
5	2.50	1.00	1.00	0.50				
6	3.00	1.00	0.50	1.50				
7	2.00	0.25	0.25	0.25	1.25			
8	2.50	1.00	1.50					
9	3.25							
10	3.00	0.75	0.50	1.75				
11	2.50	1.50	1.00					
12	1.50	0.50	0.50	0.50				
13	3.25	2.25	1.00					
14	4.00	2.00	2.00					
15	2.50	2.00	0.50					
16	2.50	2.00	0.50					
17	2.00	1.50	0.50					
18	2.00							
19	1.50	1.00	0.50					
20	2.50	0.50	0.50	1.50				
21	3.75	1.00	2.75					
22	2.75	2.25	0.50					
23	2.50							

TOTAL	59.50

EXAM 8 FALL 2015 SAMPLE ANSWERS AND EXAMINER'S REPORT

GENERAL COMMENTS

- Overall candidates considered this to be a long, difficult exam but one that was of much higher quality than the 2014 exam. The Syllabus & Exam Committee selected a pass mark that reflects candidates' actual performance in light of the length of the exam.
- Candidates commented in depth on the length of some of the material in some questions. The Syllabus & Exam Committee strives to strike a balance between the length of a question as written vs. how clear a question is. 'Long' questions (i.e. more than one page of material) are usually reserved for more complex questions where the committee wants to provide more clarifying information to make it clearer to candidates the answer and analysis we are expecting. The Syllabus & Exam Committee will continue to refine our timing of the exam, taking into account the time necessary to read and digest material within a given question.
- Many candidates lost credit or lost time by not focusing on what the question was asking candidates to do. Often times we received lengthy answers of questions where the stem was 'Briefly describe' which wastes exam time for no additional benefit. We also received many short, insufficient answers to questions where the stem was 'Fully Describe'. The stem of the question is chosen carefully by the Syllabus & Exam Committee to inform candidates how much time and effort they should spend on a particular question. We again refer candidates to the Future Fellows article from December 2009 titled "The Importance of Adverbs" for additional information on this topic.
- For questions with multiple steps or parts that depend on answers from other parts other questions, an incorrect calculation in one part did not impact any credit applied for subsequent parts.
- On this exam, the committee asked a few questions that were either from newer exam material (ex. Question #21), questions that had not been asked before (ex. Question #3), or new and novel questions on frequently tested material (ex. Question #13). The committee notes that performance on these questions was **materially** worse than performance on the exam overall. The committee continues to stress to candidates that they should be referring to and studying from the source readings in order to understand the concepts being tested.

EXAM STATISTICS

Number of Candidates: 771
Available Points: 59.5
Passing Score: 40.75
Number of Passing Candidates: 313
Raw Pass Ratio: 40.60%
Effective Pass Ratio: 42.18%

QUESTION 1

Total Point Value: 2.5 **Learning Objective: A2.B**

Sample Answers

Part a: 0.5 points

Using earned car years may create maldistribution because some territories (or other non-merit rating variables) may have higher frequency. But using premium assumes the high frequency is reflected in higher premium and territorial differentials are proper, however, state regulation prevents territorial rating therefore, territorial differentials are not proper and premium is not necessarily a better exposure base.

Part b: 1.0 points

Number of Accident-Free Years	Earned Car Years	Number of Claims Incurred	Frequency
1 or More	700,000	35,000	0.050
C = 0	100,000	9,000	0.090
Total	**800,000**	**44,000**	0.055

Mod = Z * R + (1 – Z)

<u>For one or more year's accident-free:</u>

Mod = 0.05 / 0.055 = 0.909; R = 0;

- ⇨ 0.909 = 1 – Z
- ⇨ Z = 0.0909;

Part c: 1.0 points

Current Average Claim Frequency = 0.055 (44,000 / 800,000)

Mod = Z * R + (1 – Z)

<u>Since prior claim experience follows Poisson distribution and average claims is non-zero:</u>

Mod = 0.09 / 0.055 = 1.636; R = $\frac{1}{(1-e^{-\lambda})}$, where λ = current average claim freq = 0.055;

- ⇨ 1.636 = 18.686*Z + (1 – Z)
- ⇨ Z = 0.036;

EXAMINER'S REPORT

Part a:

- Most candidates mentioned the maldistribution that exists using car-years as an exposure base.
- A common error among candidates was arguing that earned premium is preferred since it corrects for maldistribution that exists due to territorial differences. Candidates failed to realize this was not an advantage since territorial rating is prohibited; hence territorial differentials are not proper.
- Some candidates argued premium may still be a stronger exposure base if non-territorial factors are captured correctly therefore reducing the maldistribution that exists using car-year – this was given full marks.

Part b:

- Candidates performed very well on this subpart; The majority of the candidates received full credit.

Part c:

- Candidates performed relatively well on this subpart; many candidates got full credit.
- Some common errors are:

i) Incorrect formula for R = $\frac{1}{(1-e^{-\lambda})}$;

  a. Common incorrect formulas:

    i. $R = \frac{1}{(1+e^{-\lambda})}$

    ii. $R = \frac{\lambda}{(1+e^{-\lambda})}$

ii) Incorrect calculation for λ:

  a. Many candidates used the 0-year frequency (.09) instead of the total frequency (.055).

iii) Incorrect calculation of the Modification (there was no common error that was made in the calculation of the Modification)

QUESTION 2

Total Point Value: 2.75 **Learning Objective: A3**

Sample Answers

Part a: 1.00 points

Sample 1

Intrinsic aliasing is occurring because there are covariates for each level of each variable used. By definition Beta 4 = 1 – Beta 1 – Beta 2 – Beta 3 and Beta 8 = 1 – Beta 5 – Beta 6 – Beta 7.

Extrinsic aliasing is occurring for vehicle class truck and territory C because all observations in C are trucks and vice versa. Based on nature of data, Beta 3 = Beta 7.

These can lead to convergence issues or confusing results. Alternatively, modern GLM software will usually automatically correct for these.

Part b: 0.50 points

Sample 1

Near aliasing occurs when there is strong correlation (but not perfect) between covariates. In our case, Territory D is highly correlated with Other type vehicle class. This will cause convergence issues.

Part c: 1.25 points

Sample 1

- First I would eliminate all observations with Territory = D and Vehicle Class = Car.
- We can eliminate Truck (Beta 7) and Other (Beta 8) because they are aliased with Territory C and D, respectively.
- From there I would eliminate Vehicle Class = Van so the model is uniquely defined.
- So we have Beta 1, Beta 2, Beta 3, Beta 4, and Beta 5 for a total of 5 covariates.

Sample 2

- We can have a base term for Territory A and class Car (eliminates intrinsic aliasing)
- We can keep Territory C and eliminate class Truck (eliminates extrinsic aliasing)
- We can eliminate the 30 cars in Territory D (eliminates near aliasing) and get rid of Other class

- Covariates are Beta 0 (A and car), Beta 2 (Territory B), Beta 3 (Territory C), Beta 4 (Territory D), Beta 6 (Class Van).
- 5 covariates needed.

Examiners Report

Part a:

Candidates did reasonably well on this part of the problem, generally receiving a majority of the possible points. The candidate was expected to identify that intrinsic aliasing was present as the fourth territory (or vehicle class) could be expressed as a linear combination of the other territories (or classes). Extrinsic aliasing is present because Territory 'C' and vehicle class 'Trucks' are perfectly correlated – that is, all trucks are in territory C and territory C is comprised of only trucks.

Both types of aliasing can result in convergence issues as the model will not be uniquely defined. Another acceptable response is that for both types of aliasing, modern GLM software will make the necessary corrections.

When only partial credit was given, common mistakes included:

- Simply stating that intrinsic aliasing was caused because there were 8 parameters without identifying the linear relationships between them.
- Failing to provide examples of intrinsic and extrinsic aliasing using the data provided.
- Failing to describe the impact of the intrinsic or extrinsic aliasing.

Part b:

In general candidates did well on this part of the problem, with a majority of candidates receiving full credit. The candidate was expected to highlight that Territory D and the 'Other' vehicle class were nearly perfectly correlated and that this near aliasing would create convergence issues, unstable parameter estimates, or confusing results. The most common mistake in this part of the problem was failing to provide both the example and impact as requested in the problem.

Part c:

This subpart proved to be the most difficult as few candidates received full credit. This part was challenging in that multiple instances of aliasing needed to be addressed to receive full credit. To receive full credit the candidate needed to:

- Remove either Beta 3 or Beta 7 to address the extrinsic aliasing between Territory 'C' and vehicle class 'Truck'
- Address the rogue data causing the near aliasing (the 30 cars in territory D) by reclassifying or removing the observations
- Remove either Beta 4 or Beta 8 as Territory 'D' and vehicle class 'Other' are now extrinsically aliased after removing the 30 cars in Territory 'D'

- To address the intrinsic aliasing, the candidate needed to remove one additional parameter or remove two parameters (one territory and one vehicle type) and introduce an intercept.
- This will result in a total of 5 covariates.

Common mistakes included:

- Failing to address the rogue observations causing the near aliasing
- Including too many parameters in the intercept (e.g., including all of Territory 'C', Territory 'D', and classes 'Truck' and 'Other' in the intercept)

QUESTION 3

Total Point Value: 2.5 **Learning Objective: A3**

Sample Answers

Sample 1

Need Form: $Y = g^{-1}\,(\beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3) + \varepsilon$

$g(x) = \log(x)$

$g^{-1}(x) = e^{\beta x}$

We have from the four observations:

$0.092 = e^{\beta_1+\beta_3}$

$0.0267 = e^{\beta_1}$

$0.15 = e^{\beta_2+\beta_3}$

$0.05 = e^{\beta_2}$

Loglikelihood: $\ell = \sum_{i=1}^{4} -\mu_i + y_i \ln \mu_i - \ln(y_i!)$

$$= -(e^{\beta_1+\beta_3}) + 0.092 \ln\left(e^{\beta_1+\beta_3}\right) - \ln(0.092!)$$
$$- (e^{\beta_1}) + 0.0267 \ln\left(e^{\beta_1}\right) - \ln(0.0267!)$$
$$- (e^{\beta_2+\beta_3}) + 0.15 \ln\left(e^{\beta_2+\beta_3}\right) - \ln(0.15!)$$
$$- (e^{\beta_2}) + 0.05 \ln\left(e^{\beta_3}\right) - \ln(0.05!)$$
$$= -(e^{\beta_1+\beta_3}) + 0.092(\beta_1 + \beta_3) - \ln(0.092!)$$
$$- (e^{\beta_1}) + 0.0267(\beta_1) - \ln(0.0267!)$$
$$- (e^{\beta_2+\beta_3}) + 0.15(\beta_2 + \beta_3) - \ln(0.15!)$$
$$- (e^{\beta_2}) + 0.05(\beta_2) - \ln(0.05!)$$

$$\Rightarrow \frac{d\ell}{d\beta_1} = -\left(e^{\beta_1+\beta_3}\right) + 0.092 - e^{\beta_1} + 0.0267 = 0$$
$$= -\left(e^{\beta_1+1.149}\right) + 0.092 - e^{\beta_1} + 0.0267 = 0$$
$$\Rightarrow \left(e^{\beta_1+1.149}\right) + e^{\beta_1} = 0.1187$$
$$\left(e^{1.149} \cdot e^{\beta_1}\right) + e^{\beta_1} = 0.1187$$
$$e^{\beta_1} = 0.02857$$
$$\beta_1 = -3.555$$

Expected frequency of Male state A= $e^{\beta_1+\beta_3} = e^{-3.555+1.149} = \mathbf{0.09}$

Sample 2

$$\begin{array}{l} MA \\ MB \\ FA \\ FB \end{array} \begin{pmatrix} 0.092 \\ 0.0267 \\ 0.15 \\ 0.05 \end{pmatrix} = g^{-1}\left[\begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}\right]$$

After $g^{-1}()$ => right side becomes = $\begin{pmatrix} e^{\beta_1+\beta_3} \\ e^{\beta_1} \\ e^{\beta_2+\beta_3} \\ e^{\beta_2} \end{pmatrix} = \vec{\mu}$

$$\ell(y;\mu) = \sum_{i=1}^{4} -\mu_i + y_i\, ln(\mu_i) - ln(y_i!)$$

$$= -(e^{\beta_1+\beta_3}) + 0.092(\beta_1+\beta_3) - ln(0.092!)$$
$$-(e^{\beta_1}) + 0.0267(\beta_1) - ln(0.0267!)$$
$$-(e^{\beta_2+\beta_3}) + 0.15(\beta_2+\beta_3) - ln(0.15!)$$
$$-(e^{\beta_2}) + 0.05(\beta_2) - ln(0.05!)$$

ignore all constant terms, $-\ln(y_i!)$

$$\frac{d\ell}{d\beta_1} = 0 = -e^{\beta_1+\beta_3} + 0.092 - e^{\beta_1} + 0.0267$$
$$0 = -e^{\beta_1+1.149} + 0.092 - e^{\beta_1} + 0.0267$$
$$0.1187 = e^{\beta_1}(1+e^{1.149})$$
$$\beta_1 = -3.5555$$

$$\frac{d\ell}{d\beta_2} = 0 = -e^{\beta_2+\beta_3} + 0.15 - e^{\beta_2} + 0.05$$
$$0.2 = e^{\beta_2}(1+e^{1.149})$$
$$\beta_2 = -3.0338$$

	State A	State B
M	$e^{-3.5555+1.149} = \boxed{0.09013}$	$e^{-3.5555} = 0.02857$
F	$e^{-3.0338+1.149} = 0.1519$	$e^{-3.0338} = 0.04813$

Examiners Report

This question tested the candidate's ability to formularize and solve a Generalized Linear Model. With the CAS's continued focus on advanced analytics, the process and execution of solving a GLM is something that candidates are expected to know and is included within the Learning Objective on the syllabus.

One common mistake was that candidates used the SSE instead of a loglikelihood function when plugging in the covariates. In the case of a Classical Linear Model, solving the GLM produces results that are identical to those derived when minimizing the Sum of Squared Errors for a CLM. Because this was not a CLM, the SSE could not be used to solve the problem.

Candidates generally did well identifying the X and β matrices, but struggled relating these two matrices to the Y matrix. Candidates should be aware of the relationship between these 3 matrices.

Candidates struggled to identify and apply the log link function correctly. Partial credit was given if the loglikelihood function was used but an incorrect link function was used with the covariates.

Candidates generally knew to take the partial derivative of the loglikelihood function with respect to each β and set these partial derivatives to 0 in order to solve for the β's. Candidates did not need to solve for β_2 to receive full credit for the problem, and points were not taken off for solving for this parameter.

Common mistakes included:

- Not identifying and applying the log link function correctly
    - Incorrectly stating the relationship between the 3 matrices (multiplying the X and β matrices together, without using the correct $g^{-1}(x)$ function, and setting equal to the Y matrix)
    - Using the logit link function instead of the log link function
    - Not recognizing that μ needed to equal $e^{\Sigma X\beta}$. Candidates frequently set μ equal to $\sum X\beta$ in the loglikelihood function
- Changing the β's given in the problem, including:
    - Adding β_0, or some intercept term
    - Removing a β that was given. There were 4 factors in the problem and 3 covariates were given. These did not need to be adjusted, since there was no aliasing in the problem
- Using the SSE instead of the given loglikelihood function
- Not stating that the partial derivatives with respect to each β need to be set equal to 0
- Calculation errors with derivatives, exponentials and ln()

  - For example, $e^{\beta_1+1.149} + e^{\beta_1} \neq e^{2\beta_1+1.149}$
- Using the incorrect formula for the expected frequency for a Male in State A

QUESTION 4

Total Point Value: 2.25 **Learning Objective: A1**

Sample Answers

Part a: 1.5 points

Sample 1

Chi-square test statistic: $X^2 = \sum_i \frac{(Ai-Ei)^2}{Ei}$

The null hypothesis H_0, is that the claim frequency is <u>not</u> shifting over time

Actual # claims=exposures * frequency

Expected # claims = exposures * λ

Year	Actual	Expected
2010	104.5	114
2011	110	132
2012	169	156
2013	126	126
2014	120	144

Test statistic is $(104.5\text{-}114)^2$ / 114 + … + $(120\text{-}144)^2$ / 144 = 0.792 + 3.667 + 1.033 + 0 + 4 = 9.542

Since TS > critical value of 9.49, I would reject the null hypothesis and say that claim frequency is shifting over time.

Sample 2

Ho: Frequency is not shifting over time

$$X^2 = \sum w \frac{(freq - \lambda)^2}{\lambda}$$

X^2= $9500(.011\text{-}.012)^2/.012 + 11000(.01\text{-}.012)^2/.012 + 13000(.013\text{-}.012)^2/.012 + 10500(.012\text{-}.012)^2/.012 + 12000(.01\text{-}.012)^2/.012 = 9.542$

9.542>9.49, reject Ho ➔Frequency is shifting

Part b: 0.75 points

Sample 1

Lagged Year Correlations:

1. Calculate the average correlation in frequency for each one year lag (2009-2010, 2010-2011, etc)
2. Do this same average correlation calculation for two year lag, three year lag, etc.
3. If the average correlation decreases as lag increases, frequency is likely shifting over time.

Sample 2

Another method would be a correlation test. For every pair of lag years, for example 1 year lag (2000-2001, 2002-2003, etc.) Calculate the correlation between frequencies of those years and average all the correlations of the pairs to get the 1 year lag correlation. Do this for all lags, like 2-year lag (2000-2002, 2001-2003, etc.), 3-year lag (2000-2003, 2001-2004, etc.), etc. If the correlation decreases as the lag increases, then can conclude that the parameters are shifting over time.

Examiners Report

Part a:

Crucial in part A was setting up the test by stating the hypothesis, noting the formula for the Chi-square statistic, calculating the items in the formula & the statistic, and reaching the correct conclusion by comparing the calculated statistic to the given critical-value by declaring whether you accept or reject the hypothesis.

The question specifically asked for candidates to state their hypotheses, calculate the test statistic, and state their interpretation of the test result. Most candidates calculated the test statistic and stated a conclusion, but many candidates forgot to state the null hypothesis. Some candidates did not show sufficient work and it was difficult to determine knowledge of the material if a wrong test statistic was calculated or a wrong conclusion was drawn.

If the candidate made a calculation error, but was able to draw the proper conclusion based on that error, they got credit for their conclusion, but not the calculation. For example, if the candidate miscalculated the test statistic to something smaller than 9.49 and accepted their null hypothesis, they got credit for the conclusion; and if they miscalculated to something larger than 9.49 and rejected their null hypothesis, they got credit for the conclusion. This assumes that the candidate stated the hypothesis correctly.

Part b:

This part was looking for the correlation test described by Mahler on page 235, 4th paragraph. Most candidates were able to do this, some in a few sentences, some a little longer. Key words sought were “correlation”, “pairs of years”, “average correlation for all pairs with same lag”, “correlation decreases as lag increases”. We did see a handful of alternate answers, but most of these did not meet the “fully describe” statement in the question.

Where candidates lost credit, they were generally unclear in describing that they were calculating the corrections between pairs, or did not mention that they should take the average of all correlations between pairs of the same lag. It was not enough to say that correlation changes, or that it decreases over time. Neither gives the impression that the candidate knew that the correlation decreases as lag increases. We saw answers using terms like "difference", "variance", "covariance", and “confidence interval" instead of "correlation". We also saw answers that implied a correlation among more than a pair of years.

QUESTION 5

Total Point Value: 2.50 **Learning Objective: A2a, A2c**

Sample Answers

Part a: 1.00 points

Sample 1

$$E[w_2] = E[W] + b_w\,(V_2 - E[V]) + c_w\,(W_2 - E[W])$$

$$= \underbrace{\frac{20+25}{2000+2000}}_{\text{Component 1}} + \underbrace{0.2\left(\frac{3+2}{1000+1000} - \frac{5+3}{2000+2000}\right)}_{\text{Component 2}} + \underbrace{0.3\left(\frac{10+13}{1000+1000} - \frac{20+25}{2000+2000}\right)}_{\text{Component 3}}$$

$$= 0.011425$$

Part b: 1.00 points

Sample 1

Given a class, group into quintiles by predicted class frequency. Compute the sum of squared errors compared to the holdout sample value for the following 3 predictions: credibility procedure, raw data, hazard group relativity. If the credibility procedure is effective, it should have the lowest sum of squared errors of the three.

Sample 2

Derive ratios for all classes using credibility procedure. Rank all classes from smallest to largest by credibility relativity. Group into quintiles and calculate relativity of quintile ratio to the hazard group ratio for credibility estimate, new estimate, and holdout estimate. Calculate SSE for credibility, raw, and hazard group vs the hold out estimates. Method with the lowest SSE is best approach.

Part c: 0.50 points

Sample 1

There is too much noise in the individual test. Grouping into quintiles diversifies away the class specific variation allowing one to see the effect of the credibility procedure.

Sample 2

Each class is relatively small compared to the hazard group and results can be volatile from class to class, grouping into quintile allows for a more credible evaluation of the results.

Examiners Report

Part a:

Generally, candidate responses fell into one of four categories:

1) Those who followed the prescribed methodology exactly and arrived at the correct final answer.
2) Those who used only one of the two years provided and followed the methodology correctly.
3) Those who used the weights in the problem to arrive at a reasonable estimate but didn't follow the methodology.
4) Those who did not successfully attempt the problem.

Those in group 1 received full credit. Those in groups 2 and 3 received various degrees of partial credit depending upon how far along they were with the calculations. Those in group 4 received minimal partial credit.

Part b:

Most candidates attempted to describe the quintiles test as it applies to the calculation of the multidimensional credibility weighted relativities.

To receive full credit, the candidate should have at the minimum provided the following:

- Sort the credibility weighted relativities.
- Group the classes into quintiles based upon the sorted relativities with similar number of TT claims.
- Calculate the relativities for the three basic methods (credibility, raw, hazard group) by quintile. (This can be shown with formulas of SSE)
- Calculate the SSE for each method against the holdout data relativities, and choose method with lowest SSE.

Partial credit was given for addressing each of these steps in the quintiles test.

Some candidates addressed the quintiles test as it relates to other applications (e.g. experience mods, excess ratios). Some partial credit was given in this case.

Some candidates pointed out that there were only two classes in part a of the problem, so a quintiles test could not be performed. However, part b asked the candidate to describe how such a test would be performed. It did not ask them to calculate it from the data given in part a.

Part c:

Some candidates only addressed the shortcomings of the SSE test, but failed to address why the quintiles test was better. Some candidates only addressed the shortcomings of the quintiles test, but failed to address why the SSE test was better. In both of these cases partial credit was awarded.

QUESTION 6

Total Point Value: 3.00 **Learning Objective: A1e, A1a**

Sample Answers

Part a: 1 point

Sample 1

1. Assign each risk to a group.
2. Compute the centroid of each group.
3. Assign each risk to the group that has the closest centroid to the risk.
4. If there is a change from the previous grouping go to 1) and continue until no risks change groups.

Sample 2

Steps in K Means Clustering

First assign policies to one of k clusters arbitrarily

1) Calculate the centroid of each cluster using premium weights
2) Reassign policies to nearest centroid/cluster using L2 distance
3) If any policies get reassigned go back to step 2 and repeat until no politics are reassigned.

Part b: 0.50 points

Sample 1

The new method should be used because there is less overlap of coverage amounts for each deductible factor range.

Sample 2

I would pick proposed method since new groups have less variance within group (bars have shorter length compared to current) and more variance between groups (since there is less overlap/more spread between groups compared to current).

Part c: 1.50 points

Sample 1

Measurability – the risk characteristic (Coverage A level) is conveniently and reliability measureable. This would support implementation.

Expense – if the expense to implement is greater than the benefit gained, then the company should not implement. A factor to consider is that the number of groups remains the same (4 to 4) which will limit system change costs.

Sample 2

Availability of Coverage – because the new plan is more accurate, the insurer would be willing to write business that it otherwise wouldn't. This increases coverage availability.

Constancy – want the classification to remain constant over life of policy. For the most part, Coverage A amount is determined at the start of the policy period and should remain constant.

Examiners Report

Part a:

Most candidates earned full credit. The k-means algorithm as described in the paper has 4 steps. Graders read each response to see if what they wrote contained something that resembled each step (even if they didn't delineate the steps) and correctly iterated until termination.

If candidates listed variations or put steps out of order, graders still checked to see if the desired result would be achieved. Candidates lost credit if it looked like they missed something equivalent to one of the steps or if their version of the algorithm didn't iterate to termination correctly. Graders were flexible in what they would accept as a full credit answer.

Part b:

To earn full credit, candidates had to make the correct selection of method and provide a proper explanation or comparison of methods. Most candidates did both, but some did one or the other or else provided correct information about one of the methods but not the other, so it wasn't clear why the choice was in fact correct.

Part c:

To earn full credit, candidates had to identify two operational considerations, explain how they relate to classification plan selection, and tie them back to the particulars of the question. Common pitfalls included:

- Many candidates only identified and explained considerations without applying them to the information in the question or only identified and related the considerations without explaining what they mean, i.e. they only did two of the three things for each consideration

- For some of the considerations, candidates misunderstood the distinction between the classification and the associated rating factors. For example, lack of Constancy can be described by Coverage A shifting with inflation, not the fact that insureds are likely to change deductibles

Some candidates also did not list an Operational consideration and listed other types of considerations (ex. Statistical considerations)

QUESTION 7

Total Point Value: 2.00 **Learning Objective: C5**

Sample Answers

Part a: 0.25 points

Sample 1

$$\int_R^S G(x)dx$$

where $G(x) = 1 - F(x)$

Sample 2

$$\int_R^S S(x)dx$$

Part b: 0.25 points

Sample 1

$$\int_0^R xf(x)dx + R\,(1 - F(R))$$

Sample 2

$$\int_0^R xdF(x) + R\,(1 - F(R))$$

Part c: 0.25 points

Sample 1

$$\frac{G + H + B + C + D}{B + C + D}$$

Sample 2

$$\frac{B + C + D + G + H}{B + C + D}$$

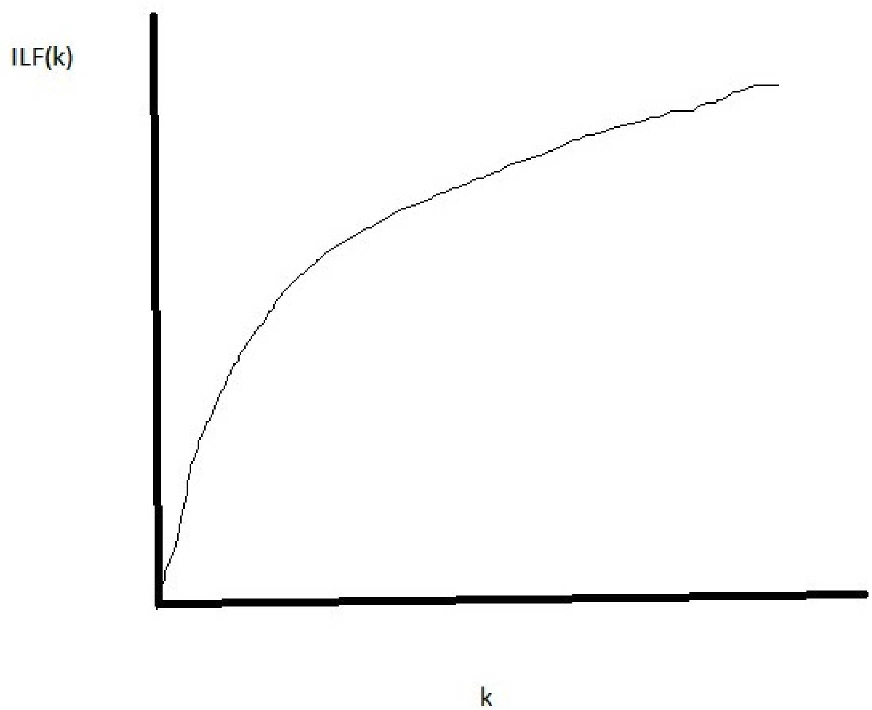
Part d: 1.25 points

Sample 1

The first derivative of the ILF is positive. ILF' >= 0

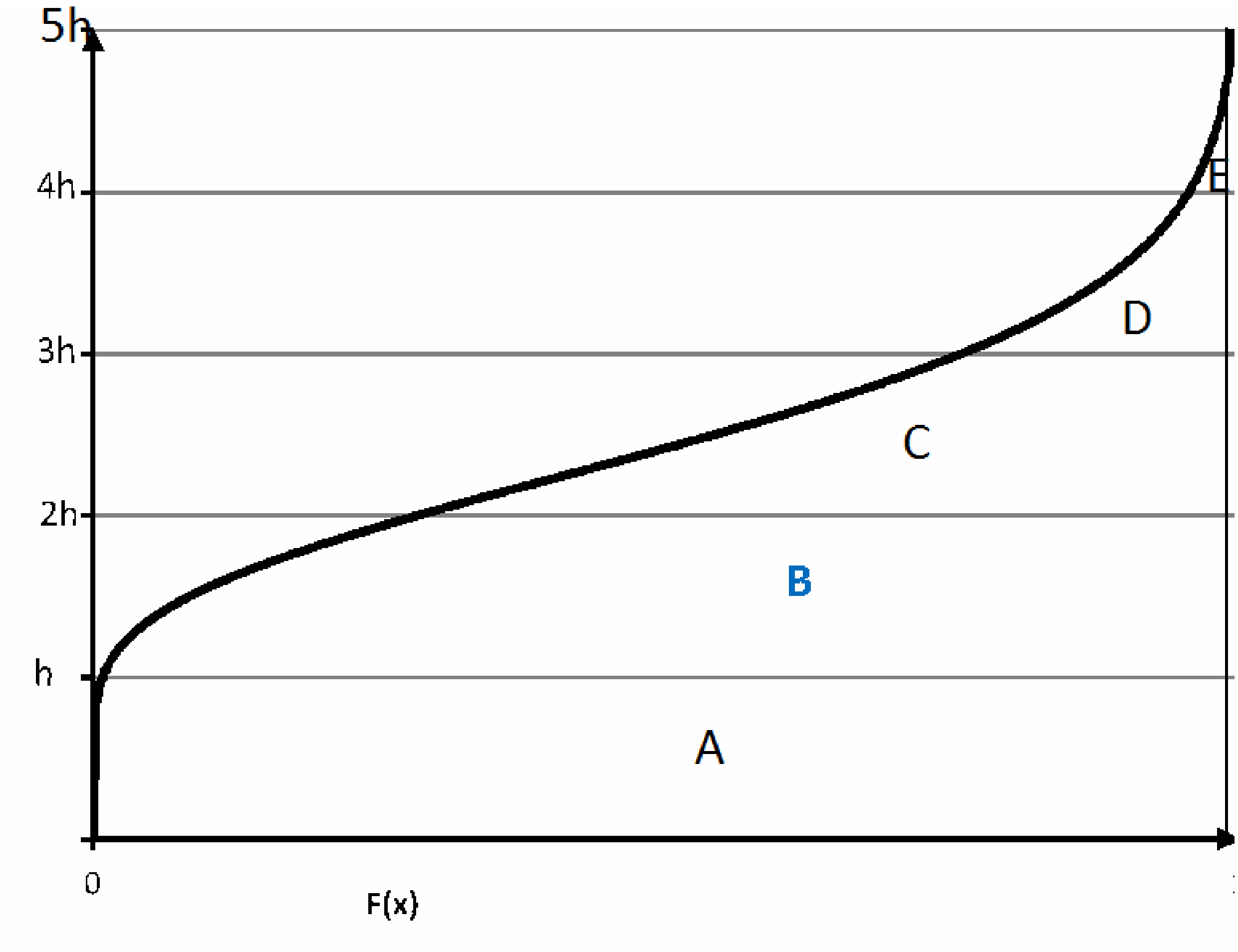
Second derivative is negative. ILF'' <= 0

ILFs increase @ a decreasing rate and thus approach a constant -> see graph below



The consistency test evaluates whether the marginal change is increasing @ a decreasing rate. The graph above shows as the limit, k, grows, the ILF increases but @ a decreasing rate

Sample 2



For constant size of intervals (height h), each successive layer is smaller than layer below it. E < D < C < B.

As one adds additional coverage, the probability of reaching higher layer is lower so marginal cost of layer is cheaper. This is why ILFs should increase at a decreasing rate, which is checked by the consistency test. Marginal cost of layer decreases with higher layers.

Examiners Report

Part a:

Most candidates performed well on this question. There was no partial credit given for this subpart given the point value. The most common error is the confusion/understanding of the definition of G(x) and F(x).

Part b:

Candidates struggled on this part of the question.. There was no partial credit given for this subpart given the point value. The candidates who missed this question mostly did not know how to set up an integral of the area tested using the size method. Some candidates got area B set up correctly and missed the equation for areas C and D.

Part c:

Most candidates performed well on this question. There is no partial credit given for this subpart given the point value. The most common error was to include area L in the numerator of the equation.

Part d:

Most of the candidates did very well stating/explaining the consistency test and got the full portion of credit for this part of the response. Some candidates lost points on the remainder of the partial credit for not fully connecting the consistency theory back to the graph. Also some candidates provided an explanation that was in the right direction but some facts were not stated correctly or accurately. For example, some candidates had equations of the marginal increase in ILF setup incorrectly/incompletely, or the description of the areas under the curve were unclear. If no explanation was given for the graph presented (tying back to the consistency theory), then some partial credit was deducted, but credit was still given for the presentation of the graph. Some candidates did not have the correct labels for the x and y axes of the graph, or the labels were switched which made the shape of the graph incorrect. When the labels were switched, very often the explanation was incorrect also, and thus the candidate lost some points as well.

QUESTION 8

Total Point Value: 2.5 **Learning Objective: B1, B2**

Sample Answers

Part A: 1.00 points

Sample 1

Empirical losses at higher limits may be volatile. Curve fitting can smooth out the volatility.

Empirical losses may not reach maximum policy limits, so no factor can be calculated (free cover). Curve fitting can extrapolate losses to higher limits.

Sample 2

There can be gaps in the data if the empirical data is thin at higher sizes of loss.

There can also be cluster points in the data around round numbers.

Curve fitting can smooth over cluster points and provide information where gaps occur to reduce impact from having gaps.

Sample 3

Losses used in fitting the curve may develop further. Curve fitting can take loss development (and even the dispersion in development) into consideration

The credibility at the high end of the distribution is a concern. Curve fitting fits a curve that maximizes the likelihood of all reported losses.

Part b: 1.50 points

Sample 1

- ILFs below a certain threshold can be determined directly from the data.
- It allows us to rely on the actual data for the lower layers where there is a larger volume of data subject to random fluctuations. ILFs above that threshold can be estimated using curve fitting (e.g. a simple or mixed distribution) to more accurately estimate losses at higher dollar amount intervals.
- The threshold above which curve fitting should be employed should be selected to permit the maximum reliance on reported data while still retaining enough data above the threshold to permit reasonable fitting of a loss distribution. It should be a round number prior to the 'thinning out' of the data.
- This method provides a smooth transition from relying on data for lower accident limits to relying on a fitted curve to provide some information at higher accident limits.

Sample 2

One would use empirical data where credible; cutoff at a level where there is just enough higher limit losses to fit a curve (perhaps $100k). Then we can fit an Exponential curve for the first layer of XS losses; and this will provide a smooth join to the empirical loss distribution. Finally, we can use the Pareto distribution to fit the highest layer of losses.

Sample 3

Up to a certain cutoff point ($100k), use empirical data directly with the empirical distribution. Above the cutoff point, shift and truncate the data, then fit a mixed Exponential/Pareto distribution to the shifted and truncated data. This approach allows us to use actual data as much as possible at layers where it is sufficiently credible, while preserving enough data above the truncation point to fit curves. The Exponential component of the mixture is light-tail and reflects losses just over the truncation point, while Pareto is heavy-tail and best reflects high loss layers. The shifted and truncated mixed Exponential/Pareto distribution can be smoothly joined to the empirical distribution up to the truncation point, yielding consistent results.

Examiners Report

Part a:

- In order to receive full credit a candidate needed to:
  - Identify two distinct issues with empirical ILFs that could be resolved by curve fitting
  - Provide a brief description of how curve fitting would overcome.
- Common errors made by candidates were as follows:
  - Giving two valid shortcomings but failing to describe how curve fitting would overcome the shortcomings
  - Stating only that "curve fitting solves this problem" without description of how.
- Candidates are expected to be able to draw on a list of problems with empirical ILFs that can be addressed through curve fitting.
- Note that ILFs built directly from empirical data using an approach as described in Lee would not fail the consistency test. Issues with failing consistency test were therefore not given credit.
- A few items that were noted as problems with empirical data also cause the same problem with curve fitting and can be solved in similar ways with use of empirical data. For example, selection issues related to differences in severity profile for risks purchasing different limits was often given as an issue with the solution being to make separate curves for different limits. This process is also possible using empirical ILFs by segmenting that data similarly so no credit was given for answers of this type.
- Credit was given for recognizing development as an issue with empirical data but credit was only given for how curve fitting helps if it incorporated an element of using a distribution for dispersion of development. Saying to build the curve only on mature data was not acceptable as the same solution could be applied in an empirical method.

- Issues with policy limits causing a bias in the distribution were acceptable if accompanied by an explanation that the calculation of variance for use in risk load was improved through fitting a theoretical curve.

Part b:

Candidates needed to mention selection of a truncation point below which empirical data should be used directly and above which a curve should be fit. The solution should explain why this is done. Most candidates failed to identify that the empirical distribution and curve should be joined smoothly together.

Fully illustrating a mixed Exponential/Pareto curve without addressing reliance on empirical data below the truncation point received no credit.

QUESTION 9

Total Point Value: 3.25 **Learning Objective: B3**

Sample Answers

Sample 1

Basic Limits Expected Loss = Basic limits premium x Expected loss & ALAE Ratio
= 800,000 x .8
= 640,000

	(1)	(2)	(3)	(4)	(5) = (1) x (2) x (3) x (4)	(6)	(7)	(8) = (5) x (6) x (7)
Yr	BLEL	PAF_{13B}	PAF_{13C}	Detrend	CSLC	EER	LDF	Exp Dev
2012	640,000	1.0	1.0	.907	580,480	.995	.519	299,763
2011	640,000	1.0	1.0	.864	552,960	.995	.338	185,966
2010	640,000	1.0	1.0	.823	526,720	.995	.198	103,769
					1,660,160			589,498

Look up 1,660,160 to get Z = .85 EER = .995 MSL = 551,800

Large Losses	Basic Limits Loss	Basic Limits Loss & ALAE Limited by MSL	Reduction to Ground Up
June 30, 2010	100,000	551,800	648,200
Dec 31, 2011	100,000	300,000	50,000
April 5, 2012	55,000	115,000	0

Historical Loss & ALAE limited by basic limits and MSL
= 1,500,000 + 400,000 + 350,000 + 600,000 + 400,000 + 2,000,000 - 648,200 - 50,000
= 4,551,800

$$\text{AER} = \frac{\text{historical limited} + \text{expected dev}}{\text{CSLC}} = 3.097$$

$$\text{Mod} = \text{Z} \times \left(\frac{\text{AER} - \text{EER}}{\text{EER}}\right) = .85 \times \left(\frac{3.097 - .995}{.995}\right) = 1.796$$

Sample 2

-assuming no changes to exposure

	Effect date	1 Prem	2 ELR	1 x 2 BLEL	13.b	13.c	detrend	CSLC
premises/ops	3/1/12	800,000	.8	640,000	1.0	1.0	.907	580,480
	3/1/11	800,000	.8	640,000	1.0	1.0	.864	552,960
	3/1/10	800,000	.8	640,000	1.0	1.0	.823	526,720
								1,660,160

CSLC = 1,660,160

$\rightarrow$ EER = .995

Z = .85

MSL = 551,800

$\rightarrow$ calculate % unreported

	CSLC		EER		Dev factor		% unreported
2012	580,480	x	.995	x	.519	=	299,762.8
2011	552,960	x	.995	x	.338	=	185,965.98
2010	526,720	x	.995	x	.198	=	103,769.11
							589,497.9

$$\text{mod} = \frac{\text{AER} - \text{EER}}{\text{EER}} \times \text{ Z} \qquad \text{AER} = \frac{\text{actual} + \%\text{ unrptd}}{\text{CSLC}}$$

Per 5A assume basic limits = $100,000 per size MSL is 551,800

2010 = ground up = 1,500,000 limited by BL = 1.5M – 600k = 900,000
then MSL limits ALAE to get includable losses of
$1,451,800

2011 same technique limit losses by BL and total by MSL
losses = 750,000

2012 no limits from individual loss so includable = 2.35M

$$\text{AER} = \frac{1{,}451{,}800 + 750{,}000 + 2{,}350{,}000 + 589497.9}{1{,}660{,}160}$$

= 3.097

$$\text{mod} = \frac{3.097 - .995}{.995} \times .85 = 1.796$$

factor = 1 + 1.796 = 2.796

Examiner's Report

This was a straightforward but calculation intensive question which was very similar to the example shown in the ISO manual accompanying the exam. The majority of candidates performed well and demonstrated understanding of most or all of the steps needed to arrive at the final modification.

The most common mistake was to use only the large losses in the calculation of the Actual Experience Ratio, ignoring the total ground-up losses altogether.

Another common error was to omit the $100,000 basic limit when calculating the impact of large losses.

Some candidates lost partial credit when calculating the Company Subject Loss Cost, by not showing the Annual Basic Limits Premium multiplied by the Expected Loss and ALAE Ratio.

We accepted both the experience modification, and its factor form (by adding 1) as a final answer, as long as the labelling was consistent.

QUESTION 10

Total Point Value: 3.0 **Learning Objective: B3**

Sample Answers

Part a: 0.75 points

Sample 1

1, represents the manual rate
$Zp * \frac{Ap-Ep}{E}$, the charge based upon actual primary losses deviated from expected primary losses
$Ze * \frac{Ae-Ee}{E}$, the charge based upon actual excess losses deviated from expected excess losses

Sample 2

1→ unity term, no difference from class plan

$Zp * \frac{Ap-Ep}{E}$ →Primary layer mod. Primary credibility multiplied by actual primary deviation from expected.

$Ze * \frac{Ae-Ee}{E}$ →Excess layer mod. Excess credibility multiplied by actual excess deviation from expected.

Part b: 0.5 points

Sample 1

Zp is usually larger because the primary loss experience tends to be more stable than excess loss experience and is more reflective of future loss potential.

Sample 2

Zp is typically larger because primary losses have more experience and therefore less volatility so we can assign more credibility. In contrast, excess losses are more rare and have a longer tail, which increases their volatility and therefore we should allow less credibility

Part c: 1.75 points

Sample 1

(1) $0 \leq Z \geq 100\%$
(2) $\frac{d}{dx}Z \geq 0$

(3) $\frac{d}{dx} Z/_x < 0$

Function 2: 105% > 100% violates (1)
Function 4: credibility decreases as expected loss increases, violates (2)
Function 1: $\frac{35\%}{2000} > \frac{15\%}{1000}$ violates (3)
Function 3: satisfies (1), (2), (3) Most appropriate

Sample 2

a) Credibility should be:
(1) $0 \leq Z \geq 1$
(2) $\frac{dZ}{dE} \geq 0$
(3) $d\frac{Z}{E}/_E < 0$

Function	Commentary
1	X – no, does not satisfy #2 (see below)
2	X – no, credibility > 1 for 5000 size (violates #1)
3	Y – yes, select this one
4	X – no, credibility decreases as size increases; violates #2

	Marginal Rate	
EL	Function 1	Function 3
1000		
2000	.0002 = (.35-.15)/1000	.00008 = (.63-.55)/1000
3000	.0002	.00007
4000	.0002	.00006
5000	.0002	.00005

Not decreasing

Examiners Report

- **Part a:** Most candidates did well on part a. The question was straightforward and all terms defined in the question. Candidates lost points where they failed to described the terms. Most candidates were awarded full credit, especially for the 2nd and 3rd term.
    - Several candidates noted only that the first term allows the modification to be multiplicative. This response did not demonstrate enough understanding of the material since it failed to relate this term back to its role in reflecting the manual rate as a starting point. This response was not given credit.

- Some candidates commented that the second term represented frequency and the third term represented severity. Unless other comments were included, points were not awarded for frequency. Frequency suggests that the comparison was based upon claim counts and not loss amounts and was not considered correct unless further described to represent loss dollars.
- A handful of candidates failed to describe the role of the 1 (unity factor) at all.

Part b: Most candidates were awarded full credit for part b responses. Almost all candidates were able to identify that Zp > Ze, but not all were able to explain why.

- Some candidates stated that there was more data within the primary. Without further explanation as to why more data should mean more credibility, full credit was not awarded.
- A minor number of candidates quoted the maximum value of Zp and Ze in the current experience rating. This again did not explain why Zp was larger and was not awarded full credit.
- Similarly, a handful of candidates noted the Zp must be greater as w = Ze/Zp and w <1. Again, this did not explain why and was not awarded full credit.

Part c: Again, most candidates were awarded full credit for part c responses. As shown in the sample responses, the candidate did not have to show their calculations to receive full credit. However, candidates who reached incorrect conclusions and did not show work did not receive partial credit.

- Several candidates commented about the range of values for Function 1 being more appropriate than Function 3, or that the credibility awarded under Function 3 for E[L] = 5000 was too high. This was subjective; objective criteria or explanation was required. As a result, candidates who solely referred to the range of values and not the linear slope of Function 1 vs. the decreasing slope of Function 3, were not awarded full points.
- A minor portion of candidates interpreted the E[L] to represent the value of a single claim, and Z the credibility to a single claim of that value. The information provided in the question did not support this interpretation and it was not accepted as a valid answer.

QUESTION 11

Total Point Value: 2.50 **Learning Objective: B4b**

Sample Answers

Part a: 1.50 points

Sample 1

Risk	Mod	Manual LR	Modified LR
1	.9	75%	83.3%
2	1.4	115%	82.14%
3	.7	55%	78.6%
4	1.2	95.6%	79.63%
5	.8	115%	81.25%

Risk	Mod	Manual LR	Modified LR
3	.7	55%	78.6%
5	.8	115%	81.25%
1	.9	75%	83.3%
4	1.2	95.6%	79.63%
2	1.4	115%	82.14%

There is good dispersion in the manual loss ratios as the mod increases so the plan is good at identifying risks. There is no clear trend in the standard loss ratios so the plan is good in correcting differences. Overall, the plan is effective.

Part b: 1.00 points

Sample 1

I would want risk and class characteristics as this would indicate if the mod was correcting for a poor class fit. I would want the risk's loss history as this would indicate if a random large loss (or a few) is driving the high mod.

Sample 2

Prior loss history that goes into the experience mod. It is possible that risk 4 had one large, random loss that is increasing its mod but isn't actually predictive of future losses. May want to limit large losses if this is the case.

Recent changes to risk 4 that could impact losses. For example, if risk 4 just implemented a new return to work program the future losses would be expected to be lower than the experience suggests.

Examiners Report

Part a:

Overall, candidates scored well on this subpart and successfully calculated the mods, manual loss ratios and modified loss ratios. Most candidates were also able to use the calculated values to conclude that the experience rating plan was effective. Comments on the manual loss ratio trends and the lack of trends in the modified loss ratios were required for full credit.

Part b:

Candidates performed well on this subpart although not as well as they did on part a. To obtain full credit, candidates needed to successfully identify two *separate* pieces of information and explain its impact on the mod.

QUESTION 12

Total Point Value: 1.50 **Learning Objective: B4**

Sample Answers

Part a: 0.5 point

Sample 1

Necessary condition: credit or debit mods should have equal standard loss ratios in the prospective period.

The debit/credit mod standard LRs are fairly close, thus I'd say the condition is met.

Sample 2

The necessary condition for credibility says that debit and credit risks should have the same permissible loss ratio.

The ratios are not the same, so the plan does not satisfy this condition.

Sample 3

Should equally reproduce the permissible LR. By size, each group seems to be relatively close.

Small	1.05 vs. 1.08
Medium	0.98 vs. 0.96
Large	0.99 vs. 1.00

Could do better for small risks, slightly better for medium risks, but overall meets necessary condition.

Sample 4

Necessary: debit and credit risks should generate same LR.

For total LR: $\frac{1.05+0.98+0.99}{3} \neq \frac{1.08+0.96+1.00}{3}$

Risks with credit mod ≠ risks with debit mod (assuming LR is evenly distributed among the 3 risk sizes)

So necessary condition is not met.

Within each risk size, LR for credit vs. debit are also not the same.

Part b: 0.5 point

Sample 1

Efficiency test statistic = variance(modified LR)/variance(unmodified LR)

Risk Size	Test Statistic
Small	.008/.07 = 0.114
Medium	.004/.05 = **0.08**
Large	.004/.04 = 0.1

Medium risk has the **lowest** statistic, so it has the most accurate experience rating based on the efficiency test.

Sample 2

Medium risk has best experience rating as (modified variance)/(unmodified variance) is lowest so it's the best improvement based on experience rating (0.004/0.05).

Sample 3

Efficiency test statistic = variance(manual LR)/variance(standard LR), **higher** is better

Small	.07/.008 = 8.75
Medium	**12.5**
Large	10

Medium risks have most accurate experience rating.

Sample 4

Efficiency test: [var(unmodified) – var(modified)]/var(unmodified)

Small	(.07-.008)/.07 = 0.8857
Medium	**0.92**
Large	0.9

Medium is the most accurate.

Part c: 0.5 point

Sample 1

It's better to correct with the manual rates, because an <u>overall</u> inadequacy is best handled in the base rates. Experience mod plan is intended to adjust for <u>individual</u> cost differences.

Sample 2

Changing manual rates will be better to correct for inadequacy because if the experience rating plan adjusts for the inadequacy (off-balance increases) then the problem will persist. The off-balance will increase and mask some of the actual rate level needed.

Sample 3

Since many of the small risks will not qualify for the experience rating plan, this is better corrected through manual rates.

Sample 4

It is better to correct the manual rates. Applies to new policies without experience, so no need to wait for experience before premium is accurate.

Examiner's Report

Part a:

- Candidates were expected to understand the necessary condition and how to apply it to the given data.
- Candidates scored very well on part a, with the majority receiving full credit.
- Common mistakes included confusing the sufficient condition with the necessary condition, and comparing standard loss ratios between size groups rather than credit/debit mod groups.

Part b:

- Candidates were expected to know how to conduct the efficiency test and interpret the results.
- Candidates scored very well on part b, with the majority receiving full credit.
- A common mistake was making calculation errors that lead to an incorrect conclusion.
- Candidates who calculated the efficiency test statistic correctly but drew the incorrect conclusion received partial credit.
- Candidate that made small math errors in the calculation but drew the right conclusion received full credit as they were able to demonstrate knowledge and understanding of the concept.

Part c:

- Candidates were expected to understand the purpose of experience rating as a tool to correct for individual risk differences rather than overall premium adequacy.
- Part c was more challenging for candidates, though the majority received at least partial credit.

- A common mistake was identifying the experience rating plan as the better way to correct premium inadequacy.

QUESTION 13
Total Point Value: 3.25 **Learning Objectives: B6, B7**
Sample Answers
Part a: 2.25 points

LDD:

Sample 1
At 18 months, 435,000/6.55 = 66,412 of loss below the deductible is expected to have been paid. Insured owes the insurer $66,412 as reimbursement.

Retro Policy:

Sample 1
The retro premium formula is R = (b + CL + cF)T.
F is the expected excess loss, which in this case is 650,000 - 435,000 = 215,000.
At 18 months, 435,000/3.75 = $116,000 of loss below the retro limit is expected to have been incurred.
R = (150,000 + (1.1)(116,000) + (1.1)(215,000))(1.045) = 537,235
Insured has already paid $1M in deposit premium, so the insurer owes the insured $462,766.

Sample 2
The retro premium formula is R = (b + CL + cF + cV)T.
F is the expected excess loss, which in this case is 650,000 - 435,000 = 215,000.
At 18 months, 435,000/3.75 = $116,000 of loss below the retro limit is expected to have been incurred.
Assume the insured elects to include V, the retro development. V = 435,000(1 – 1/3.75) = 319,000
R = (150,000 + (1.1)(116,000) + (1.1)(215,000) + (1.1)(319,000))=903,925
Insured has already paid $1M in deposit premium, so the insurer owes the insured $96,075.

Part b: 1.0 points
Sample 1

i. LDD is least attractive and would be most subject to credit risk, since the insurer pays all claims upfront, and then needs to recover loss amounts below the deductible from the insured. There's a chance the insured won't or can't pay.
ii. Excess policy is least attractive and would be most subject to interest rate risk because it has the longest payout period.

Examiners Report
Part a: Generally, candidates struggled with both the LDD and Retrospective cash flow calculation. Common errors for LDD were ignoring the deductible reimbursement as a portion of the cash flow and attempting to calculate the premium. Candidates should remember that workers compensation payments go directly to claimants and not the insured, thus it was not necessary for candidates to calculate claim payments made by the insurer.

Common errors for the Retro were not including the excess loss provision in the calculation and using incurred losses at ultimate as opposed to including the 18 month valuation. In some cases candidates tried to use the 1,000,000 deposit premium as the standard premium. The amount of standard premium was not required for this problem.

The provision for retro development in retro policies is an optional provision that insureds elect into. For this reason, candidates were not expected to include the development factor in their calculation. For candidates that included a provision for development in their retro premium calculation, they needed to explicitly state their assumption in order to receive full credit.

The Expected losses were supposed to be interpreted as ultimate values and candidates should've been able to recognize that. However, candidates had the potential to receive full credit if they treated the expected limited losses of 435,000 as an 18 month valuation.

Part b: Candidates generally scored well on this question, with a majority receiving full credit. In order to receive full credit on the question, candidates needed to detail which of the options were least attractive and support their choice. The most common error was detailing which plan was most attractive without giving indication to which was least attractive.

QUESTION 14

Total Point Value: 4.00 **Learning Objective: B2a, B7a&b**

Sample Answers

Part a: 2.00 points

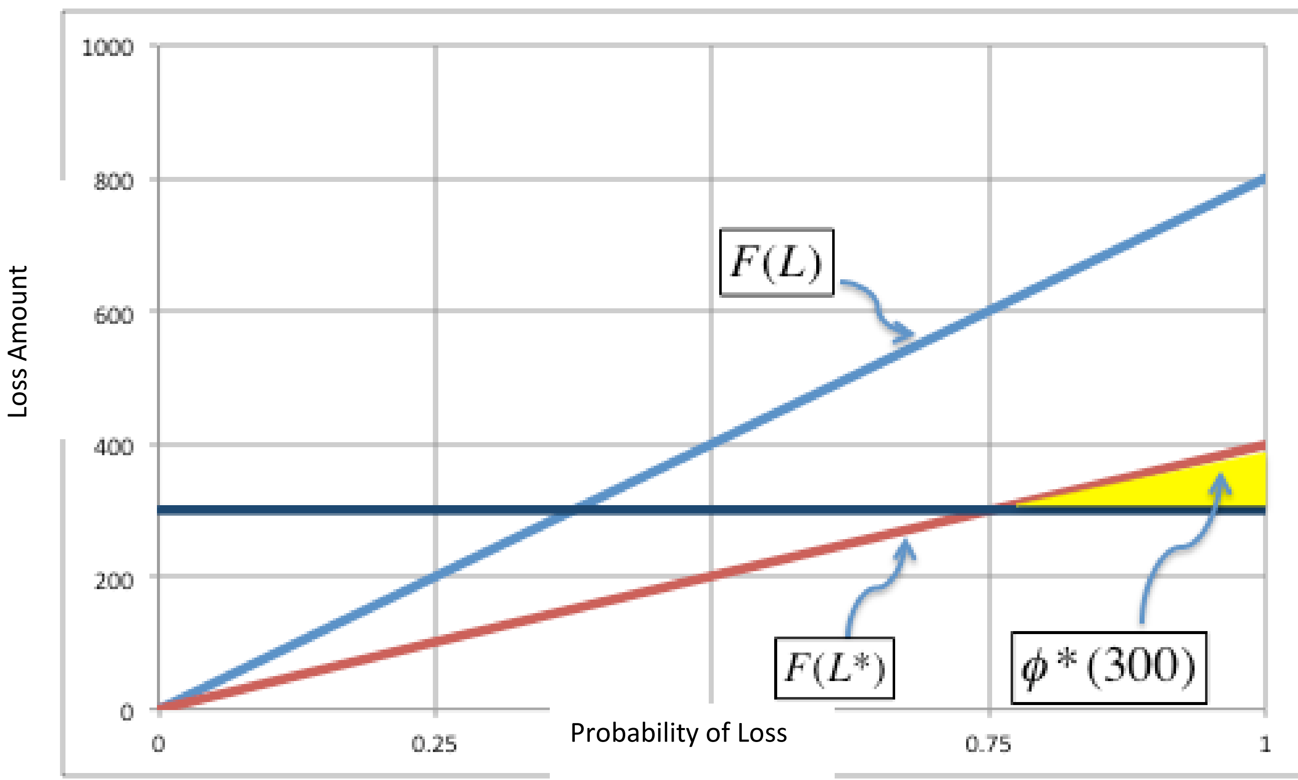
(in \$000)

Let L = total loss, and $L*$ = loss limited to deductible

Then E[L] = 400 and E[$L*$] = 200, meaning XS = 400 – 200 = 200

Draw a Lee diagram to get the insurance charge:

i



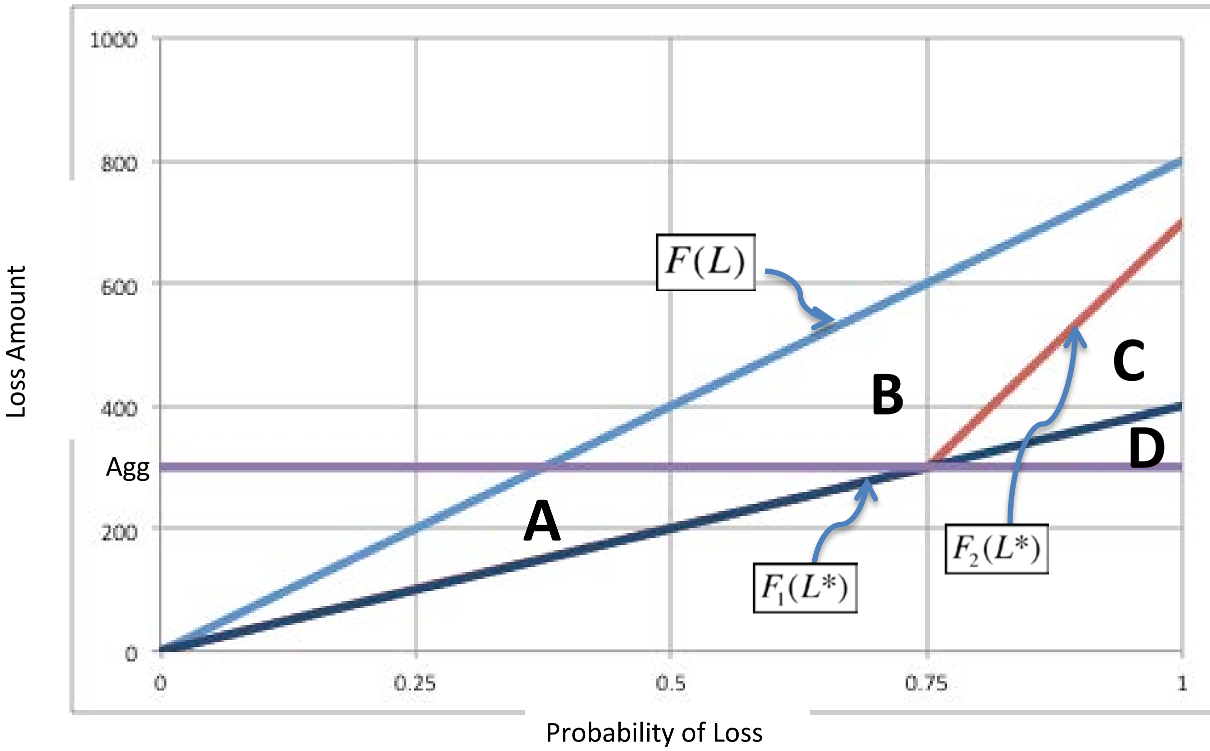
$$\phi^*(300) = (.5)(.25)(100) = 12.5$$

$$LDD = \frac{200 + 12.5 + 400(.075 + .05) + 45}{1 - .06 - .125 - .04 + .05} = 372.727 \rightarrow \$372{,}727$$

Loss retained by the insured = (.5)

Part b: 2.00 points

Sample 1:



Loss cost under old distribution = Excess Loss + Insurance Charge

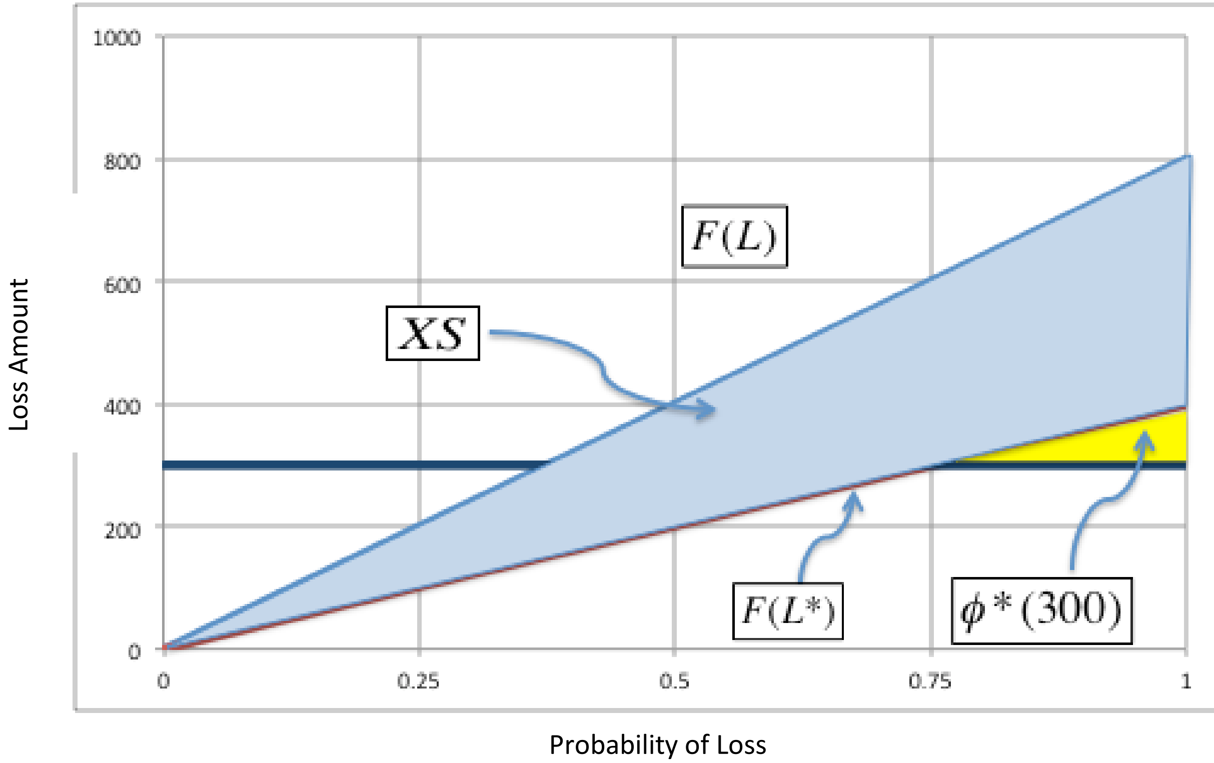
=(A+B+C)+D

Loss cost under new distribution = Excess Loss + Insurance Charge

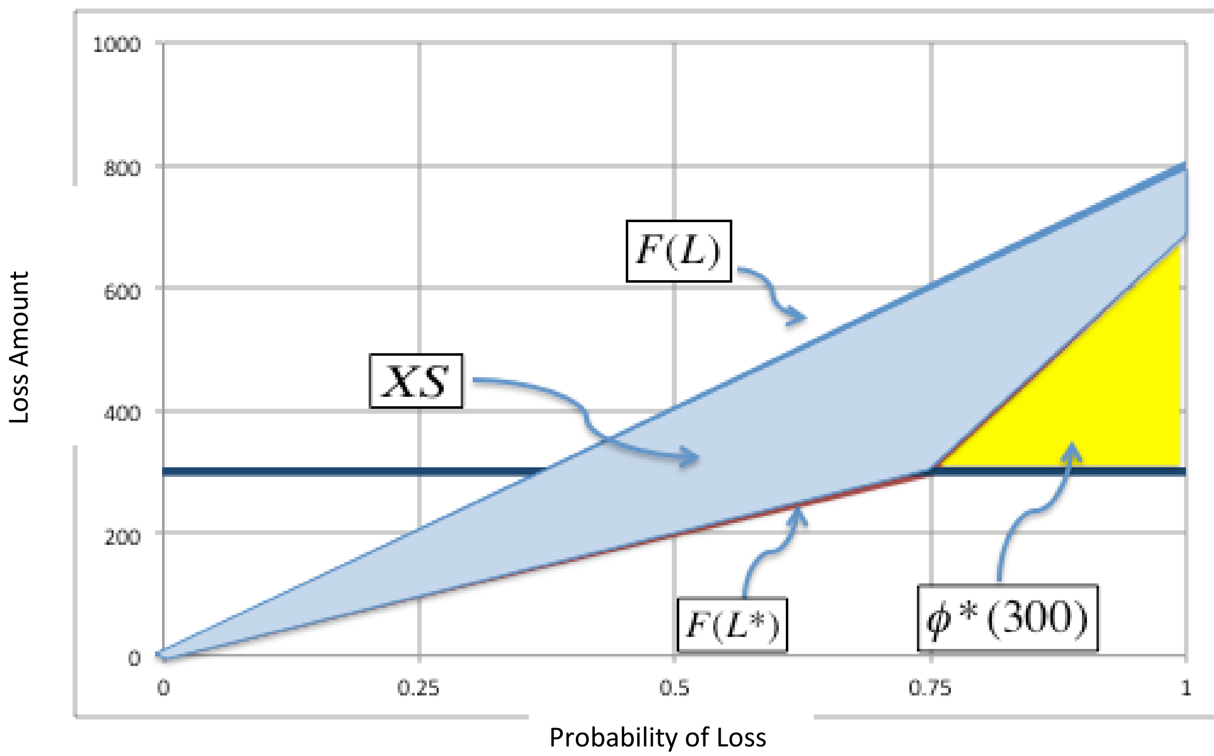
=(A+B)+(C+D)

Since loss costs are equal and expenses don't change, no change in LDD premium.

Sample 2:



Recall from Part a that expected loss cost is $200,000 + $12,500 = $215,000



New insurance charge = (.5)(.25)(700,000 – 300,000) = 50,000

Expected unlimited loss (same as Part a) = 400,000

New expected limited loss = (.5)(.75)(300,000)+(.25)(300,000)+50,000 = 237,500

New expected excess loss = expected unlimited – expected limited

= 400,000 – 237,500 = 162,500

New expected loss cost = expected excess + insurance charge

= 162,500 + 50,000 = 212,500

Since expected loss cost didn't change, expenses won't change, so same LDD premium.

Examiners Report

Part a:

Notes:

- Candidates were not required to draw a Lee Diagram for Part a.
- Several candidates calculated the limited expected loss ($187,500), and subtracted this from the total expected loss to get the Table L charge of $212,500. This was an equivalent approach to the above, and received full credit.
- A few candidates assumed that (per Teng) the 12.5% commission was included in the 6% acquisition, but this doesn't make mathematical sense. The A term in Teng's formula for LDD premium should include both the 6% and 12.5% as a total acquisition cost.
- Where candidates included the insurance charge in the formula, but then failed to calculate the charge, partial credit was still given for the numerator.

Common Errors:

- Applying ULAE and LBA percentages to limited or excess losses (instead of unlimited expected loss).
- Using limited expected loss as the expected loss cost in the numerator of the LDD premium formula.
- Separating fixed expense as an additive amount at the end instead of putting it in the numerator.
- Not including the insurance charge as part of the loss cost.
- Trying to use NCCI's Table M and the ICRLL procedure to calculate the insurance charge instead of calculating directly based on the given loss distributions.
- Where candidates developed the insurance charge as a percent of limited expected loss, several then misapplied that percentage to unlimited expected loss.

Part b:

Notes:

- The intent of the question was to produce a graphical demonstration that the LDD premium wouldn't change. Candidates who recalculated the excess and/or insurance charges for Part b were given credit **only** if these were accompanied by an accurate Lee Diagram **and** were calculated correctly.
- If a candidate produced a Lee Diagram for Part a, they didn't need to reproduce that graph for Part b in order to get credit for the original limited distribution or the unlimited distribution.
- Some candidates used the entire area of the Table L charge for their demonstration, which received full credit.
- Some candidates recalculated not only the expected loss cost, but the premium as well. If calculated correctly, this was still worth full credit.

Common Errors:

- Several candidates didn't accurately produce a graph of the new limited distribution.
- Many candidates who didn't produce a graph of the total loss distribution also failed to identify the new excess losses. Likewise, many candidates who didn't produce the aggregate limit line also failed to identify the new insurance charge.
- A common error was to identify only one change in either the excess loss or the insurance charge, but not both; or to identify both changes, but to fail to correctly identify that they completely offset each other, or comment at all on the impact to premium.

QUESTION 15

Total Point Value: 2.50 **Learning Objective: B5a**

Sample Answers

Part a: 2.00 points

Sample 1

b = e – (c – 1) * E[A] + c * I

26,820 = 20,000 - .17 * 70,000 + 1.17 * I

I = 16,000 $^{I}/_{E[A]} = {}^{16}/_{70} = .229$ $^{E[A]}/_{P} = 0.7$

LR	#	# above	% above	Charge	r (entry ratio)
0	0	5	1	.5+1*.35/.7 = 1	
.35	1	4	.8	$.3+.8*\frac{(.525-.35)}{.7}=.5$	.35/.7 = .5
.525	1	3	.6	$.15+.6*\frac{(.7-.525)}{.7}=.3$	.75
.7	1	2	.4	$.05+.4*\frac{(.875-.7)}{.7}=.15$	1
.875	1	1	.2	$0+.2*\frac{(1.05-.875)}{.7}=.05$	1.25
1.05	1	0	0	0	1.5

Charge at 1.0 = .15

.75 = Savings(.75) + 1 – Charge(.75)

.75 - 1 + Charge(.75) = Savings(.75)

.75 – 1 + .3 = Savings(.75) = .05

Charge(1.0) – Savings(.75) = .15 - .05 = .10 = $^{I}/_{E[A]}$

The net insurance charge of .229 embedded within the basic premium is higher than the competitor analysis charge of .1.

Sample 2

L_R = 70,000 LR = 0.70

L_H = 52,500 LR = 0.525

Charge I → b = e – 0.17 * (70) + cI

I = 16,000 (current)

Competitors Charge = $\frac{0+0.35+0+0.175+0}{5} = 10.5\%$

Competitors Savings = $\frac{0+0+0+0+0.175}{5} = 3.5\%$

[10.5% - 3.5%] * 100,000 = 7,000

16,000 current charge vs. 7,000 implied charge, insured has a point

Part b: 0.50 points

Sample 1

Using the charge derived from competitor analysis is not appropriate because competitors may have a different mix of business and therefore have different aggregate loss curve that would produce different and not comparable insurance charges.

Sample 2

Not appropriate, basic premium includes expenses that could vary significantly from company to company.

Sample 3

It may not be appropriate to use competitor data to price the policy due to differences in certain risk characteristics although the nature of business is the same. For example, there will be differences in operations, locations, safety programs, morale of employees which varies across companies. This will result in different loss distribution, and hence, produce different insurance charge.

Sample 4

The data of 5 risks is not credible, of much less credible than the industrywide data going into the NCCI table M. The data on the 5 risks has low credibility and is not appropriate.

Examiners Report

Part a:

The intent of this part was for the candidate to compare the net insurance charge embedded in the insured's current basic premium to the net charge from the competitor's loss ratio data (produced by building a table M using the loss ratio data or by calculating the charge and savings directly).

This required the candidate to understand the basic premium formula to back into the current insurance charge (either converted or not converted were acceptable) and also understand table M building to determine the net charge produced by the distribution of competitor data.

While building a table M was the most common approach to determining the net charge based on competitor data, there were candidates that successfully calculated the net charge using either a graphical/geometric approach or by directly calculating the charge and savings without building a table.

Common mistakes included failing to recognize the need to build a table M to determine the charge based on the competitor data, failing to draw a proper comparison between the two net charges (% of SP vs % of E), and minor mathematical errors.

Part b:

Candidates needed to identify why either the expense component or net insurance charge imbedded within basic premium might vary between the insured and competitors.

A common mistake was that many candidates identified potential differences between insured and competitor's data, but failed to make a connection as to what impact those differences would have on basic premium.

For example, only stating that mix of business could vary between insured and competitors was not sufficient to receive full credit. In this case, for full credit, the candidate would need to discuss how different mix of business would impact the aggregate loss distribution and thus the net insurance charge/basic premium.

Candidates who only stated that competitor loss ratios are volatile/small sample size did not receive credit. Candidates that commented on the small sample size/volatility in loss ratios *and* resulting low credibility in the competitor data distribution received credit.

QUESTION 16

Total Point Value: 2.5 **Learning Objective: B5b**

Sample Answers

Part a: 2.00 points

Sample 1

$Adj\ E = 4{,}000{,}000(.98) = 3{,}920{,}000$

ELG = 26.

$$\psi(r_G) = \frac{G/T - (E+e)}{cE} = \frac{12{,}300{,}000/1.025 - (4{,}000{,}000 + 1{,}629{,}440)}{(1.2)(4{,}000{,}000)} = 1.3272$$

Since $\psi(r) = \phi(r) + r - 1$, we get: $\phi(r_G) + r_G = 2.3272$

lookup in NCCI table to find r_G and $\phi(r_G)$

$r_G = 2.27$

$L_G = 2.27(4{,}000{,}000) = \mathbf{9{,}080{,}000}$

Sample 2

$Adj\ E = 4{,}000{,}000(.98) = 3{,}920{,}000$

ELG = 26.

G = (b + c*L_G)T = (b + c*E*r_G)T

b = e – (c-1)E + cE($\phi(r_G)$ - $\psi(r_H)$)

G = (e – (c-1)E + cE $\phi(r_G)$ + cEr_g) T since $\psi(r_H)$ = 0

12,300,000 = (1,629,440 – (1.2 – 1) * 4,000,000 + 1.2 * 4,000,000 $\phi(r_G)$ + 1.2 * 4,000,000 * r_g) * 1.025

2.3272 = $\phi(r_G)$ + r_g

lookup in NCCI table

r_G of 2.27 and $\phi(r_G)$ of .0572 satisfies.

$L_G = 2.27(4,000,000) = \mathbf{9,080,000}$

Sample 3

$Adj\ E = 4,000,000(.98) = 3,920,000$

ELG = 26.

H = bT

$r_G - r_H = \frac{G - bT}{cET} = r_G = \frac{12.3 - 1.025b}{1.2*4*1.025} = 2.5 - \frac{b}{4.8}$

$X_H - X_G = \frac{e + E - bT/T}{cE} = 1 - X_G = \frac{1.62944 + 4 - b}{1.2*4} = 1.1728 - \frac{b}{4.8}$

$1 - X_G - r_G = 1.1728 - 2.5$

$X_G + r_G = 2.3272$

lookup in NCCI table

$r_G = 2.27$

$X_G = .0572$

$L_G = 2.27(4,000,000) = \mathbf{9,080,000}$

Part b: 0.5 points

Sample 1

b = e – (c-1)E + cE($\phi(r_G)$ - $\psi(r_H)$)

$b = 1,629,440 - (.2)(4,000,000) + 1.2(.0572 - 0)(4,000,000)$
$= \mathbf{1,104,000}$

Sample 2

$G = (b + cL_G)T$
$12,300,000 = [b + 1.2(9,080,000)]1.025$
$b = \mathbf{1,104,000}$

Sample 3

$$(E+e)T=(b+cE(L))T$$
$$(E+e)T=(b+c(E-I))T$$
$$(5,629,440)1.025=(b+1.2(4,000,000-(.0572)(4,000,000)))1.025$$
$$5,770,176=(b+4,525,440)1.025$$

b = **1,104,000**

Examiners Report

Part a. was a difficult question for candidates, and very few received full credit. By far the most common mistake candidates made was assuming that "no specified minimum premium" implied H = 0. In a plan with no specified minimum premium, H = bT.

The simplest way to solve the problem was to use the special case balance equation from the Gillam and Snader paper when there is no specified minimum premium:

$$\psi(r_G)=\frac{G/_T-(E+e)}{cE}$$

Very few candidates, however, were able to recall this formula. Without knowledge of the formula, the problem could still be solved through alternate methods—for instance, by substituting the formula $b=e-(c-1)E+c(\phi(r_G)-\psi(r_H))E$ into the formula $G=(b+cEr_G)T$. However, few candidates were able to make this connection.

While part a proved difficult, many candidates were still able to receive full credit for part b by using the formula $b=e-(c-1)E+c(\phi(r_G)-\psi(r_H))E$ and plugging in the $\phi(r_G)$ they computed in part a (even if $\phi(r_G)$ was incorrect, full credit could be awarded for part b if all other variables and calculations were correct).

Other common mistakes included:

- Looking up the incorrect ELG in the NCCI tables
- Calculating the adjusted E of 3,920,000 in order to determine the ELG, but then using 3,920,000 in all subsequent calculations involving E
- Applying the tax multiplier to the formula $b=e-(c-1)E+c(\phi(r_G)-\psi(r_H))E$
- Not knowing how to do a lookup with r_G and $\phi(r_G)$

QUESTION 17

Total Point Value: 2.00 **Learning Objective: B5**

Sample Answers

Part a: 1.50 points

Sample 1

$E + e = (1 - D)SP$, so need to find E and e

$E = (.5)(1,000,000) = 500,000$

$$\phi(r_G) = \frac{\left(\frac{1,000,000 - 750,000}{2}\right)\left(\frac{1,000,000 - 750,000}{1,000,000}\right)}{500,000} = .0625 \text{ or } \$31,250$$

$$\psi(r_H) = \frac{\left(\frac{80,000}{2}\right)\left(\frac{80,000}{1,000,000}\right)}{500,000} = .0064 \text{ or } \$3,200$$

$b = e - (c-1)E + cI = e - (c-1)E + cE\big(\emptyset(r_G) - \psi(r_H)\big)$

$83,660 = e - .2(500,000) + (1.2)(.0625 - .0064)(500,000)$

$e = 150,000$

$E + e = (1 - D)SP \Rightarrow 500,000 + 150,000 = (1 - D)(700,000)$

$D = 0.0714 = \mathbf{7.14\%}$ **($50,000)**

Sample 2

$E[R] = b + cE[L] = (1 - D)SP$, so need to find E[L] to get D

$E = (.5)(1,000,000) = 500,000$

$$\phi(r_G) = \frac{\left(\frac{1,000,000 - 750,000}{2}\right)\left(\frac{1,000,000 - 750,000}{1,000,000}\right)}{500,000} = .0625$$

$$\psi(r_H) = \frac{\left(\frac{80,000}{2}\right)\left(\frac{80,000}{1,000,000}\right)}{500,000} = .0064$$

$$E[L] = E(1 - \emptyset(r_G) + \psi(r_G)) = 500(1 - 0.625 + 0.0064) = 471,950$$

$$E[R] = 83,660 + 1.2(471,950) = 650,000$$

$$(1 - D)(700,000) = 650,000$$

$D = 0.0714 = \mathbf{7.14\%}$ **($50,000)**

Sample 3

$E + e = (1 - D)SP$, so need to find $E + e$ to get D

$$\phi(r_H) - \phi(r_G) = \frac{(e + E) - H}{cE} \quad OR \quad \frac{(1 - D)SP - H}{cE}$$

$$H = 83,660 + 1.2(80,000) = 179,660$$

$$\phi(r_G) = \frac{\left(\frac{1,000,000 - 750,000}{2}\right)\left(\frac{1,000,000 - 750,000}{1,000,000}\right)}{500,000} = 0.0625$$

$$\phi(r_H) = \frac{\left(\frac{1,000,000 - 80,000}{2}\right)\left(\frac{1,000,000 - 80,000}{1,000,000}\right)}{500,000} = 0.8464$$

$$\phi(r_H) - \phi(r_G) = 0.8464 - 0.0625 = \frac{(e + E) - 179,660}{1.2(500,000)} \quad OR \quad \frac{(1 - D)SP - 179,660}{1.2(500,000)}$$

$$e + E = (1 - D)SP = (1 - D)(700,000) = 650,000$$

$D = 0.0714 = \mathbf{7.14\%}$ **($50,000)**

Part b: 0.50 points

Sample 1

In a retrospectively rated policy, the premium discount is realized as a reduction to expenses in the basic premium, whereas in a guaranteed cost policy, the premium discount is explicitly deducted from the standard premium.

Sample 2

In both retro and guaranteed cost policies, the premium discount accounts for reduction in expense ratio as premium increases. The discount is the same in a guaranteed cost policy and retrospectively rated policy when the plans are balanced.

Examiners Report

Part a:

- Many candidates received full credit on this subpart.
- Candidates were expected to be able to calculate expected loss given a uniform distribution constrained by a min/max loss amount. Successful candidates often drew a picture of the loss distribution as a reference.
- Common errors included:
    - Switching E[A], E[L] and E in the various calculations
    - Calculating E[L] incorrectly – Candidates often drew a picture of the Lee diagram. However, they would incorrectly account for the area of all the pieces under the curve.
    - Mixing dollar values and percentages in the same formula inappropriately (e.g. calculating charge and savings as a ratio to $1M and then later multiplying by $500K, or multiplying by $500K when the calculated charges are already in dollar terms).

Part b:

- Candidates were expected to understand that "premium discount" refers to the discount resulting from a reduction in fixed expense as a percentage of premium. Some candidates erroneously interpreted "discount" as a lower-than-expected premium in a retro policy due to:
    - actual losses emerge lower than expected
    - premium minimum/maximum
    - net insurance charge
- Successful candidates understood to speak to treatment in both guaranteed cost and retro policies.

QUESTION 18

Total Point Value: 2 **Learning Objectives: B2, B7**

Sample Answers

Sample 1

Risk	Total Losses	Deductible Losses - Limited to 150,000	Excess of Aggregate Deductible
1	150,000	150,000	0
2	715,000	450,000	0
3	150,000	150,000	0
4	750,000	600,000	150,000
5	250,000	150,000	0
Total	2,015,000	1,500,000	150,000

Total Expected Loss Cost = $\frac{(2{,}015{,}000-1{,}500{,}000)+150{,}000}{5} = 133{,}000$

Sample 2

Risk	Limited Loss	r
1	70,000 + 80,000 = 150,000	$\frac{150{,}000}{300{,}000}$=0.5
2	150,000+150,000+150,000=450,000	1.5
3	150,000	0.5
4	150,000+150,000+150,000+150,000=600,000	2
5	150,000	0.5
Total	1,500,000	

Expected Total Loss = $\frac{2{,}015{,}000}{5} = 403{,}000$

Expected Primary Loss = $\frac{1{,}500{,}000}{5} = 300{,}000$

$\emptyset(r) = \frac{450{,}000}{300{,}000} = 1.5$

Table M

r	% of risks above	$\emptyset(r)$

0	100%	0.5+1*0.5=1
0.5	40%	0.3+0.4*0.5=0.5
1	40%	0.1+0.4*0.5=0.3
1.5	20%	0+0.2*0.5=0.1
2	0%	0

$\emptyset(1.5) = 0.1$

Expected Loss Cost = (403,000 – 300,000) + 300,000*0.1 = 133,000

Sample 3 – Insured's Perspective

Risk	Deductible Losses - Limited to 150,000	Losses Capped at 450,000
1	70,000+80,000 = 150,000	150,000
2	150,000+150,000+150,000 = 450,000	450,000
3	150,000	150,000
4	150,000+150,000+150,000+150,000 = 600,000	450,000
5	150,000	150,000
Total	1,500,000	1,350,000

Expected Loss per Risk = $\frac{1,350,000}{5} = 270,000$

Examiners Report

Candidates did very well on this question overall with many getting full credit. Any deductions were usually due to a small math error.

Common places where candidates lost credit were to not divide the loss cost by 5, or to only calculate the expected loss cost for the per occurrence deductible, or the aggregate deductible but not both.

Some candidates calculated the loss cost from the insured's perspective. While the Examination Committee believes the question and the term 'loss cost' are unambiguous and require calculation from the insurer's perspective, this distinction was not considered to be of central importance in this case. Therefore this was accepted as an alternate solution.

QUESTION 19

Total Point Value: 1.50 **Learning Objective: B7B, B7C**

Sample Answers

Part a: 1.00 points

Sample 1

Premium = $\frac{[(EL)x(XL+ULAE)+(SP)x(GO+CR)]}{1-A-T-P}$

273,500 = $\frac{(1,000,000)(.65)(.1+.08)+(1,000,000)(.05+X)}{1-.05-.07-.05}$

$X = .06 = \mathbf{6\%}$

Part b: 0.50 points

Sample 1

In an LDD policy, the insurer services the claim from the ground up. As a result, LDD insurers compete with each other both on price and on the quality of their service contracts. In an XS policy the losses below the deductible are often serviced by a TPA or the insured themselves, as a result the XS insurers compete with each other primarily on price. This drives down the profit loads which can even sometimes be negative.

Sample 2

For LDD, insurer services claims in deductible layer, while on an excess policy, service is handled by a TPA. Because LDD insurers compete on both service and profit, profit can be higher than for XS insurers which compete only on price.

Sample 3

LDD policies have a shorter average payout period since they pay from the 1st dollar. An excess policy doesn't pay until the layer is reached, thus they have a longer period to collect investment income, resulting in a smaller profit load when targeting the same return on surplus.

EXAMINERS REPORT

Part a:

This was a straight forward calculation problem. Most candidates received full credit. The most common point deductions were for simple calculation errors.

Part b:

The key word in this part was to **explain.** Common deductions were for responses that specified that large dollar deductible plans (LDD) have a higher profit margin compared to excess plans, but did not give further details. Common answers were simply stating that LDD were more service based or that excess have a longer tail, but not going into the detail of why that is. We were looking for the differentiation that LDD insurers service claims from ground up where as excess insurers don’t get involved until the deductible has been breached.

QUESTION 20

Total Point Value: 2.50 **Learning Objective: C5**

Sample Answers

Part a: 0.50 points

Sample 1

First estimate the total expected pure premium under the underlying business. Then apportion the pure premium between the reinsurer and ceding company by using exposure curves to the appropriate layer of each party

Sample 2

Use the expected loss ratio to turn premium to losses, then use the exposure curve to allocate losses to the cedant and the reinsurer.

Part b: 0.50 points

Sample 1

p=0.03= $\frac{G'(1)}{G'(0)} = \frac{\frac{-b^1 \ln(b)}{(1-b)}}{\frac{-b^0 \ln(b)}{(1-b)}}$ = b

Sample 2

This is a special case of MBBEFD where bg= 1, p= 0.03, g = 1/.03 = 33.33, b=0.03

Part c: 1.50 points

Total Risk Premium = Gross Premium x Expected Loss Ratio = 6000 x 0.60 = 3,600

Ceded Risk Premium = Total Risk Premium x Exposure Factor

Exposure Factor = Ceded Risk Premium / Total Risk Premium = 2705/3600=0.7514

Exposure Factor = $G(\frac{Retention+Limit}{MPL/\ Insured\ Value})$ - $G(\frac{Retention}{MPL/InsuredValue})$ =

$G(\frac{150{,}000+Limit}{5{,}000{,}000})$ - $G(\frac{150{,}000}{5{,}000{,}000})$ = $G(\frac{150{,}000+Limit}{5{,}000{,}000})$ - $G(\frac{150{,}000}{5{,}000{,}000})$ = G(y) – G(0.03) =

G(y) - $\frac{1-0.03^{0.03}}{1-0.03}$ = G(y) – 0.1029 = 0.7514 → 0.7514+0.1029=0.8543 = G(y) = $\frac{1-0.03^{y}}{1-0.03}$

$0.97 \times 0.8543 = 0.8287 = 1 - 0.03^y \rightarrow 0.03^y = 0.1713 \rightarrow y\ln(0.03)=\ln(0.1713) \rightarrow$

$-1.7644 = -3.5066y \rightarrow y=0.5032 = \frac{150{,}000+Limit}{5{,}000{,}000} \rightarrow$ Limit = 2,365,859

Examiners Report

Part a:

This question required candidates to identify that they were allocating risk premiums between the ceding company and the reinsurer by using exposure curves.

Most candidates received full or partial credit, the most common error was to not identify that gross premium would be allocated as a result of the risk premium/loss exposure that the ceding company and reinsurer were covering.

Part b:

This question required candidates to calculate b which could have been approached a few ways. Candidates could have derived G(x) and used the formula $p=0.03=\frac{G'(1)}{G'(0)}$ to solve for b. They could have also recognized that this was a special case of the MBBEFD curves such that p=b=.03. Most candidates were able to recognize this and derive G(x).

Part c:

There were two common errors that occurred on part c. The first error involved candidates incorrectly calculating the exposure factor by dividing the ceded risk premium by gross premium rather than the risk premium ($6,000 x 60%). The second error involved candidates forgoing the subtraction of the $150,000 retention at the end of the problem to get the limit of the non-proportional reinsurance treaty.

Question 21

Total Point Value: 3.75 **Learning Objective: C3a**

Part a: 1 Point

Sample 1

Simulation	Ceded Premium	Ceded Loss	Profit Commission
1	9.8M	0.05 x 700M = 35M	Max(0, (.35 x 9.8M) – 35M = 0
2	9.8M	0	Max(0, (.35 x 9.8M) – 0 = 3.43M
3	9.8M	0.018 x 700M = 12.6M	Max(0, (.35 x 9.8M) – 12.6M = 0
4	9.8M	0.006 x 700M = 4.2M	Max(0, (.35 x 9.8M) – 4.2M) = 0
5	9.8M	0.001 x 700M = .7M	Max(0, (.35 x 9.8M) - .7M = 2.73M

Expected profit commission payable at end of 2016 = (0 + 3.43M + 0 +0 + 2.73M) / 5 = 1.232M

Sample 2

Loss amount at 65%: 700M x (.65) = 455M

Loss amount at 70%: 700M x (.7) = 490M

Simulation	Ceded Premium	Ceded Loss	Profit Commission
1	9.8M	490M – 455M = 35M	Max(0, (.35 x 9.8M) – 35M = 0
2	9.8M	0	Max(0, (.35 x 9.8M) – 0 = 3.43M
3	9.8M	468M – 455M = 13M	Max(0, (.35 x 9.8M) – 13M = 0
4	9.8M	459M – 455M = 4M	Max(0, (.35 x 9.8M) – 4M) = 0
5	9.8M	456M – 455M =1M	Max(0, (.35 x 9.8M) - 1M = 2.43M

Expected profit commission payable at end of 2016 = (0 + 3.43M + 0 +0 + 2.43M) / 5 = 1.172M

Part b: 2.75 points

Sample 1

Simulations	Terminate 2016?	Total Ceded Premium	2017 Ceded Loss	Total Ceded Loss	Profit Commission
1	No	20.3M	0.03 x 750M = 22.5M	35M + 22.5M = 57.5M	Max(0, (.35 x 20.3M) – 57.5M = 0
2	Yes	-	-	-	3.43M
3	No	20.3M	0.005 x 750M = 3.75M	12.6M + 3.75M = 16.35M	Max(0, (.35 x 20.3M) – 16.35M = 0
4	No	20.3M	0.003 x 750M = 2.25M	4.2M + 2.25M = 6.45M	Max(0, (.35 x 20.3M) – 6.45M = 0.655M
5	Yes	-	-	-	2.73M

Expected profit commission for the full term contract = (0 + 3.43M + 0 + 0.655M + 2.73M) = 1.363M

Sample 2

Loss amount at 65%: 750M x (.65) = 487.5M

Simulations	Terminate 2016?	Total Ceded Premium	2017 Ceded Loss	Total Ceded Loss	Profit Commission
1	No	20.3M	510M – 487.5M = 22.5M	35M + 22.5M = 57.5M	Max(0, (.35 x 20.3M) – 57.5M = 0
2	Yes	-	-	-	3.43M
3	No	20.3M	491M – 487.5M = 3.5M	13M + 3.5M = 16.5M	Max(0, (.35 x 20.3M) – 16.5M = 0
4	No	20.3M	490M – 487.5M = 2.5M	4M + 2.5M = 6.5M	Max(0, (.35 x 20.3M) – 6.5M = 0.605M
5	Yes	-	-	-	2.43M

Loss amount at 70%: 750M x (.7) = 525M

Expected profit commission for the full term contract = (0 + 3.43M + 0 + 0.605M + 2.43M) = 1.293M

Examiners Report

Many comments from candidates seemed to indicate that the question was focused on testing the profit commission concept by creating a very difficult profit commission question. In actuality, this problem comes from the recently added section of the Clark paper (section 5B), which focuses on the topic of alternative risk products. As finite and alternative risk reinsurance contracts can be somewhat complex, the question writers and Syllabus & Exam Committee gave much more information to candidates in the problem than we would typically in order to help candidates understand a complicated reinsurance contract and calculate the correct contract financials.

It is our practice to generally avoid questions that run onto more than one page, but in this case the Syllabus & Exam Committee felt it was better to have a longer problem that was clearer on a complicated concept than a shorter problem that would potentially be subject to different interpretations.

Part a

To receive full credit, candidates were expected to calculate the ceded losses and profit commission by simulation. The expected profit commission should be calculated as an average profit from all 5 simulations.

The most common errors were:

- Calculating ceded losses by multiplying the ceded loss ratios by loss instead of premium
- Obtaining the average ceded loss for all simulations and determining the profit commission based on that.

Part b

To receive full credit, candidates must have been able to determine which simulations would have terminated after the 2016 year. Partial credit was given for candidates that correctly applied the ceded premium, ceded loss, and profit commission formulas but failed to accurately determine which simulations were terminated in 2016.

The most common errors were:

- Incorrectly determining which simulations were terminated in 2016.
- Calculating ceded loss amounts by treating 2016 and 2017 as a combined policy year.
- Using only 2017 ceded premium and loss to determine profit commission.
- Calculating ceded losses by multiplying the ceded loss ratios by loss instead of premium.
- Obtaining the average ceded loss for all simulations and determining the profit commission based on that.
- Not including the profit commission from the 2016 terminated policies in the calculation of full term expected profit commission.

QUESTION 22

Total Point Value: 2.75 **Learning Objectives: C1b , C1c**

Sample Answers

Part a: 2.25 points

Sample 1

Aggregate loss	Event combination	Probability of combination	Exceedance probability	F(x)
60 M	1,2,3	(0.1)(0.05)(0.02) = 0.0001	0	1
50 M	2,3	(1-0.1)(0.05)(0.02) = 0.0009	0.0001	0.9999
45 M	1,3	(0.1)(1-0.05)(0.02) = 0.0019	0.0001 + 0.0009 = 0.001	0.999
35 M	3	(1-0.1)(1-0.05)(0.02) = 0.0171	0.001 + 0.0019 = 0.0029	0.9971
25 M	1,2	(0.1)(0.05)(1-0.02) = 0.0049	0.0029 + 0.0171 = 0.02	0.98
15 M	2	(1-0.1)(0.05)(1-0.02) = 0.0441	0.02 + 0.0049 = 0.0249	0.9751
10 M	1	(0.1)(1-0.05)(1-0.02) = 0.0931	0.0249 + 0.0441 = 0.069	0.931
0 M	-	(1-0.1)(1-0.05)(1-0.02) = 0.8379	0.069 + 0.0931 = 0.1621	0.8379

Sample 2

Loss	Exceedance probability
60	0
50	0.1 X 0.05 X 0.02 = 0.0001
45	0.0001+0.02 X 0.05 X (1-0.1) = 0.001
35	0.001 + 0.02 X 0.1 X (1-0.05) = 0.0029
25	0.0029 + 0.02 X (1-0.1)X(1-0.05) = 0.02
15	0.02 + 0.1 X 0.05 X (1-0.02) = 0.0249
10	0.0249 + 0.05 X (1-0.1) X (1-0.02) = 0.069
0	0.069 + 0.1 X (1-0.05) X (1-0.02) = 0.1621

Most common errors

Some candidates did not identify all the possible discrete outcomes (only identified 10M, 15M and 35M as possible aggregate losses). Points were deducted for each missing outcome.

Other candidates did not multiply by (1 – probability that event occurs) for each total loss amount.

Some candidates mistakenly used $1-\pi(1-p_i)$ instead of $\sum(p_i)$.

Part b: 0.5 points

Sample 1

0.86 lays between 0.8379 and 0.931 on the F(x) distribution → 10 M loss occurs

Sample 2

1-0.86 = 0.14

0.14 lays between 0.069 and 0.1621 on the exceedance distribution → 10 M loss occurs

Most common error

Some candidates adequately identified the probability range in which the simulation falls, but interpolated with the distribution to derive the aggregate loss rather than selecting the appropriate loss amount given the discrete distribution ($10M)

QUESTION 23

Total Point Value: 2.5 **Learning Objectives: C3b**

Sample Answers

Sample 1

Accident Date	Loss XS 1M	LDF	Developed losses in layer
July 2012	2.85	1.01	2.8785
February 2013	1.16	1.05	1.218
August 2013	0.07	1.05	0.0735
March 2014	2.12	1.1	2.332
November 2014	0	1.1	0
			6.502

LC 3 xs 1M 6.502/(5.2+5.7+5.9) = .387

Using the exposure to prevent free cover

	LC	Exposure Factor
3 xs 1	0.387	.63-.3=.33
6 xs 4	.387(.27/.33)=.317	.9-.63=.27
	.704	

.704 loss cost as % of prem

<u>Alternate</u>

Experience only goes up to ~ $4M

Use experience for 1M --> 4M layer

and exposure rating for 4M-->10M layer

Loss	AY	XS trended loss in 3M xs 1M layer	XS dev factor	dev trended loss
1	2012	2,850K	1.01	2,878,500
2	2013	1,160K	1.05	1,218,000

3	2013	70K	1.05	73,500
4	2014	2,120K	1.1	2,332,000
5	2014	0	1.1	0
				6,502K

$$\frac{\$4M}{total\ IV = \$10M} = 40\%$$

$$\frac{E_{100\%} - E_{10\%}}{E_{40\%} - E_{10\%}}$$

$$\frac{\text{Exposure in 6M xs 1M layer}}{\text{Exposure in 3M xs 1M layer}} = \frac{90\%-30\%}{63\%-30\%} = 1.8182$$

estimated loss cost

$$= \frac{6{,}502{,}000*1.8182}{5.2M+5.7M+5.9M} = 70.37\%$$

Note: candidates who selected prorated layers other than 3xs1 and 6xs4 were also given credit as long as selection was reasonable and/or justified.

EXAMINERS REPORT

Common mistakes that candidates made were as follows:

- Not developing losses.
- Not using or incorrectly selecting excess layer losses (ex. subtracting the deductible off of the cumulative annual losses and not from each occurrence)
- Other minor calculation errors
- Calculating the 38.7% experience rate but leaving that as final answer so not identifying any free cover issue
- Applying exposure rate to upper layer but not using experience rate on lower layer and applying exposure rate layer relativity for upper layer loss cost
- Incorrectly selecting exposure rates for the layers.